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MANIPAL INSTITUTE OF TECHNOLOGY

A Constituent Institute of Manipal University, Manipal

**I SEM M. Tech. (CAAD) DEGREE END SEMESTER EXAMINATIONS**
**NOVEMBER 2017**

**SUBJECT: SOLID MECHANICS (MME 5101)**
**REVISED CREDIT SYSTEM**

Time: 3 Hours. Max. Marks: 50

**Instructions to Candidates:**
- Answer **ALL** questions.
- Missing data, if any, may be assumed appropriately.

1. a) Show in a general 3-Dimensional displacement that if the displacement components in the x, y and z directions are u, v and w respectively, the strain components are as follows: **[05]**

$$\epsilon_{xx} = \frac{\partial u}{\partial x} \qquad \epsilon_{yy} = \frac{\partial v}{\partial y} \qquad \epsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \qquad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \qquad \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

b) The state of stress at a point is characterized by the components,

**$\sigma_x$ = 12.31 MPa, $\sigma_y$ = 8.96 MPa, $\sigma_z$ = 4.34 MPa**

**$T_{xy}$ = 4.20 MPa, $T_{yz}$ = 5.27 MPa, $T_{zx}$ = 0.84 MPa**

Find the values of principal stresses and their directions. **[05]**

2. a) State and discuss the maximum elastic energy theory of failure and obtain the equation for estimating the elastic energy stored in a body subjected to three dimensional state of stress. **[05]**

b) Verify whether the following strain field satisfies Saint-Venant's equations of compatibility, if ***p*** is a constant: **[05]**

$$\varepsilon_{xx} = py,\ \varepsilon_{yy} = px,\ \varepsilon_{zz} = 2p(x+y)$$

$$\gamma_{xy} = p(x+y),\ \gamma_{yz} = 2pz \text{ and } \gamma_{xz} = 2pz$$

3. a) Discuss Mohr's Circles for the 3-Dimensional state of stress in a solid and highlight the important observations. **[05]**

   b) The displacement field for a solid is given by, **[05]**

   $$\boldsymbol{u = [(x^2 + y^2 + 2)i + (3x + 4y^2)j + (2x^3 + 4z)k]10^{-4}}$$

   Determine:

   i) The state of strain at a point **P(4, 3, 2)** in the solid

   **ii)** The strain field in the direction of **PQ** having direction cosines $\mathbf{n_x = 0}$, $\mathbf{n_y = -0.447}$ and $\mathbf{n_z = 0.897}$

   iii) Direction of **P'Q'** after deformation of the solid

4. a) Given the stress-strain relations in three dimensional Cartesian coordinate reference frame for isotropic materials in the form **[06]**

$$\epsilon_{xx} = \frac{1}{E}\left[\sigma_x - \vartheta\left(\sigma_y + \sigma_z\right)\right] \qquad \epsilon_{yy} = \frac{1}{E}\left[\sigma_y - \vartheta(\sigma_x + \sigma_z)\right]$$

$$\epsilon_{zz} = \frac{1}{E}\left[\sigma_z - \vartheta\left(\sigma_x + \sigma_y\right)\right] \qquad \gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

obtain the stress-strain relations in the form

$$\sigma_x = \frac{E}{(1+\vartheta)(1-2\vartheta)}\left[(1-\vartheta)\epsilon_{xx} + \vartheta\epsilon_{yy} + \vartheta\varepsilon_{zz}\right]$$

$$\sigma_y = \frac{E}{(1+\vartheta)(1-2\vartheta)}\left[\vartheta\epsilon_{xx} + (1-\vartheta)\epsilon_{yy} + \vartheta\epsilon_{zz}\right]$$

$$\sigma_z = \frac{E}{(1+\vartheta)(1-2\vartheta)}\left[\vartheta\epsilon_{xx} + \vartheta\epsilon_{yy} + (1-\vartheta)\epsilon_{zz}\right]$$

$$\tau_{xy} = \frac{E}{(1+\vartheta)(1-2\vartheta)}\left[\left(\frac{1-2\vartheta}{2}\right)\gamma_{xy}\right]$$

$$\tau_{yz} = \frac{E}{(1+\vartheta)(1-2\vartheta)}\left[\left(\frac{1-2\vartheta}{2}\right)\gamma_{yz}\right]$$

$$\tau_{zx} = \frac{E}{(1+\vartheta)(1-2\vartheta)}\left[\left(\frac{1-2\vartheta}{2}\right)\gamma_{zx}\right]$$

b) Determine the diameter of a ductile steel bar, if the tensile load is 30,000 N, the torsional moment is 20,000 Nm and the bending moment 20,000 Nm. Use a factor of safety N = 1.5, $\sigma_y$ = 280,000 kPa and E = 207,000 kPa. Use maximum shear stress theory. **[04]**

5. a) What is principle of superposition for forces? With an illustration discuss the principle of superposition. **[06]**

   b) A solid shaft of diameter ***d = 100***$^{0.5}$ mm fixed at one end is subjected to an axial tensile force of **10,000 N** at the other end and a torque of 50,000 Nmm at its free end. Determine the principal stresses, the octahedral shear stress and maximum shear stress at a point on the curved surface of the shaft. **[04]**