Reg.No.

MANIPAL INSTITUTE OF TECHNOLOGY
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I SEMESTER M.TECH.END SEMESTER EXAMINATIONS- NOVEMBER 2017

SUBJECT: FEM FOR THERMAL ENGINEERING [MME 5142]

Time: 3 Hours. MAX.MARKS: 50

Instructions to Candidates:

- Answer **ALL** questions.
- Missing data, if any, may be suitably assumed.

Q.1A Show by applying Galerkin's Weighted Residual Method that the finite element formulation for a general 3-D Cartesian steady state heat transfer without convective loads is given by, (07)

$$\left[\iiint\left(k_x\left[\frac{\partial N}{\partial x}\right]^T\left[\frac{\partial N}{\partial x}\right]+k_y\left[\frac{\partial N}{\partial y}\right]^T\left[\frac{\partial N}{\partial y}\right]+k_z\left[\frac{\partial N}{\partial z}\right]^T\left[\frac{\partial N}{\partial z}\right]\right)dV\right]\{T\}=$$

$$\left\{\iiint \dot{q}_g\,[N]^T\,dV\right\}+\left\{\iint\left(\dot{q}_x\hat{n}_x+\dot{q}_x\hat{n}_x+\dot{q}_x\hat{n}_x\right)[N]^T\,dA\right\}$$

Q.1B Determine the Shape functions for the four noded Isoparametric rectangular element using Lagrange's Interpolation Formula. Compute the temperature and the heat fluxes at a location (2,1) in the element as shown in Fig.1 (03)

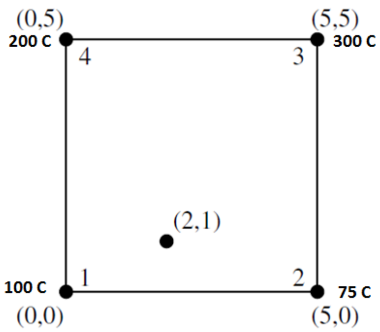

Fig 1

Q.2A What is meant by an Isoparametric element? Bring out the salient differences between Sub-parametric and Super- Parametric elements with respect to an Isoparametric element highlighting their relative importance in an FE analysis. (03)

Q.2B Starting from an assumed thermal functional for a slender one dimensional fin having the open end insulated and undergoing steady state heat transfer, apply the Variational Finite Element Formulation to obtain the Thermal Conductance Matrix and corresponding Load matrices. Assume a uniform heat generation of G W/m^3 (07)

Q.3A For the simplified uniaxial two noded fin element having conductive –convective heat transfer, by using discrete system analysis, show (with usual notations) that, (03)

$$\begin{bmatrix} \frac{kA}{L}+\frac{hPL}{4} & -\frac{kA}{L}+\frac{hPL}{4} \\ -\frac{kA}{L}+\frac{hPL}{4} & \frac{kA}{L}+\frac{hPL}{4} \end{bmatrix} \begin{Bmatrix} T_i \\ T_j \end{Bmatrix} = \begin{Bmatrix} Q_i+\frac{hPL}{2}T_a \\ -Q_j+\frac{hPL}{2}T_a \end{Bmatrix}$$

Q.3B Apply Galerkin's weighted residual formulations to obtain the stiffness conductance analogous matrix as well as the thermal load matrix for an **axisymmetric triangular thermal element.** (07)

$$[\mathrm{K}] = \frac{2\pi\bar{r}k_r}{4A}\begin{bmatrix} b_i^2 & b_ib_j & b_ib_k \\ b_ib_j & b_j^2 & b_jb_k \\ b_ib_k & b_jb_k & b_k^2 \end{bmatrix} + \frac{2\pi\bar{r}k_z}{4A}\begin{bmatrix} c_i^2 & c_ic_j & c_ic_k \\ c_ic_j & c_j^2 & c_jc_k \\ c_ic_k & c_jc_k & c_k^2 \end{bmatrix}$$

$$+\frac{2\pi hl_{ij}}{12}\begin{bmatrix} 3r_i+r_j & r_i+r_j & 0.0 \\ r_i+r_j & r_i+3r_j & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$\{\mathrm{f}\} = \frac{2\pi GA}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{Bmatrix} r_i \\ r_j \\ r_k \end{Bmatrix} - \frac{2\pi ql_{jk}}{6}\begin{Bmatrix} 0 \\ 2r_j+r_k \\ r_j+2r_k \end{Bmatrix} + \frac{2\pi hT_a l_{ij}}{6}\begin{Bmatrix} 2r_i+r_j \\ r_i+2r_j \\ 0 \end{Bmatrix}$$

Q.4A Determine the nodal Conductance Matrix and Thermal Load vector for a three noded linear triangular thermal element as given in **Fig 2** below: Given k = 20W/mm.K; thickness, t = 10 mm, All dimensions in mm (07)

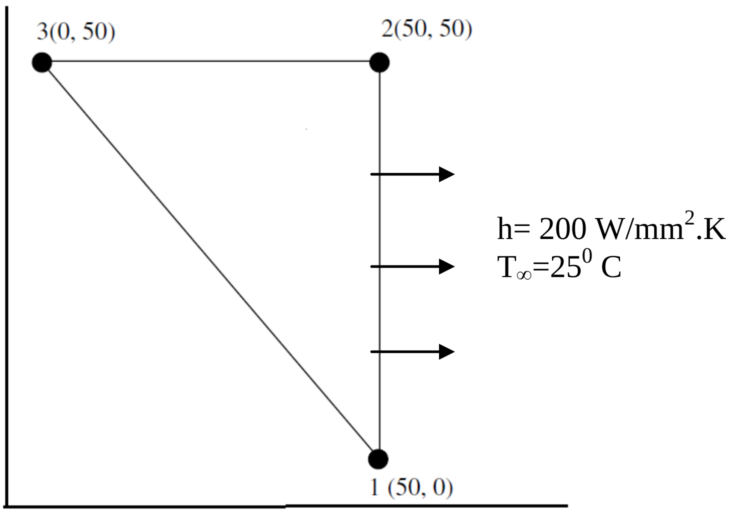

Fig. 2

Q.4B For the following thermal systems deduce their GDE using Lagrange-Euler Equations. Assume suitable functional in each case: (03)

1. One dimensional steady state heat transfer with uniform heat generation
2. One dimensional steady state heat transfer in a slender fin with convective end face.
3 Two Dimensional steady state heat transfer with surface convection

Q. 5A Derive two point sampling Gaussian Quadrature Formula. Use the same to find the value of the following Integral: Verify the same with the exact analytical answer. (03)

$$\int_{2}^{3} (2x^2 - 2e^x + 1)dx$$

Q.5B For **Unsteady state one dimensional heat conduction** in a bar subjected to uniform heat generation G W/m3, and convection over its lateral surface and subjected to end heat fluxes, derive with usual notation , the following thermal equilibrium finite element equation, (07)

$$\frac{\rho c_p l A}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{Bmatrix}\frac{\partial T_i}{\partial t}\\ \frac{\partial T_j}{\partial t}\end{Bmatrix} + \left(\frac{Ak_x}{l}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \frac{hPl}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\right)\begin{Bmatrix}T_i\\ T_j\end{Bmatrix}$$

$$= \frac{GAl}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix} - \frac{qPl}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix} + \frac{hT_a Pl}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix}$$