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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

I SEMESTER M.TECH.END SEMESTER MAKE-UP EXAMINATIONS DEC 2017

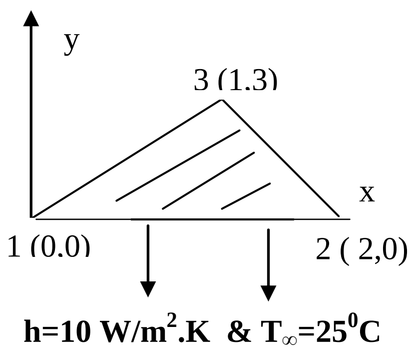
SUBJECT: FEM FOR THERMAL ENGINEERING [MME 5142]

Time: 3 Hours. MAX.MARKS: 50

Instructions to Candidates:
- Answer **ANY** questions.
- Missing data, if any, may be suitably assumed.

Q.1A Define an Isoparametric Element. Bring out the salient differences between Sub-parametric and Super- Parametric elements. Explain the relevance of each. (03)

Q.1B Determine the nodal Conductance Matrix and Thermal Load vector for a three noded Linear triangular thermal element as given in **Fig 1** below: (07)



K = 20 W/m.K
t = thickness = 10 mm

Fig. 1

Q.2A For the simplified uniaxial two noded fin element having conductive –convective heat transfer, using discrete system analysis, show (with usual notations) that, (04)

$$\begin{bmatrix} \frac{kA}{L}+\frac{hPL}{4} & -\frac{kA}{L}+\frac{hPL}{4} \\ -\frac{kA}{L}+\frac{hPL}{4} & \frac{kA}{L}+\frac{hPL}{4} \end{bmatrix}\begin{Bmatrix} T_i \\ T_j \end{Bmatrix} = \begin{Bmatrix} Q_i+\frac{hPL}{2}T_a \\ -Q_j+\frac{hPL}{2}T_a \end{Bmatrix}$$

Q.2B For the steady state radial heat conduction in a hollow cylinder, show with usual notations that the thermal conductance matrix and thermal load vectors are given respectively by, (06)

$$[K]=\frac{2\pi k}{l}\left[\frac{(r_i+r_j)}{2}\right]\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}+2\pi r_0 h\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\{f\}=hT_a 2\pi r_0\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

Q.3A Determine the temperature of the plate element shown in **Fig. 2** below, at point P. The nodal temperatures are given as T_1= 80 ^{o}C, T_2= 25 ^{o}C and T_3=40 ^{o}C. (03)

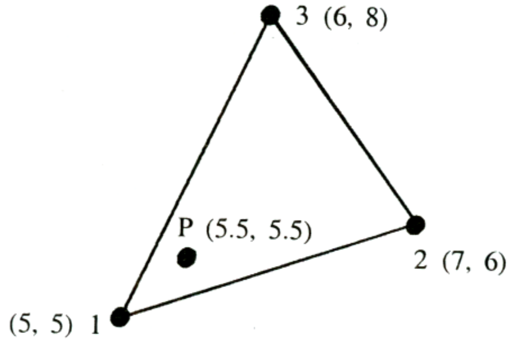


Fig. 2

Q.3B Apply Galerkin's weighted residual formulation, to obtain the Thermal Conductance and Load matrices for a slender fin with the open end insulated. Assume a two noded linear steady state heat transfer element. The Governing Differential Equation (with usual notations), is given by, (07)

$$K\frac{d^2T}{dx^2}-\left(\frac{P}{A}\right)h(T-T_\infty)=0$$

Q. 4A Derive two point sampling Gaussian Quadrature Formula. Use the same to find the value of the following Integral: Verify the same with the exact analytical answer. (03)

$$\int_3^8 (5-3e^y+2y)dy$$

Q.4B Using Variational Calculus formulation, prove that for a generall steady state conduction heat transfer, the finite element equation is given by, (07)

$$[\mathrm{K}]\{\mathrm{T}\}=\{\mathrm{Q}\}$$

where,

$$[\mathrm{K}]=\iiint_V [\mathrm{B}]^{\mathrm{T}}[\mathrm{D}][\mathrm{B}]dV+\iint_{S_2} \mathrm{h}[\mathrm{N}]^{\mathrm{T}}[\mathrm{N}]ds$$

$$\{Q\}=\iiint_V \dot{q}_g[\mathrm{N}]^{\mathrm{T}}dV+\iint_{S_2} q[\mathrm{N}]^{\mathrm{T}}ds+\iint_{S_3} \mathrm{hT}_\infty[\mathrm{N}]^{\mathrm{T}}ds$$

Q.5A Construct the finite element equations using the discrete system analysis for steady state heat transfer through the composite wall as shown in Fig.3 The following data may be used to compute the nodal temperatures. (06)

Areas: $A_1 = 2.0\,m^2$, $A_2 = 1.0\,m^2$ *and* $A_3 = 1.0\,m^2$.
Thermal conductivity: $k_1 = 2.00\,W/mK$, $k_2 = 2.5\,W/mK$ *and* $k_3 = 1.5\,W/mK$.
Heat transfer coefficient: $h = 0.1\,W/m^2\,K$
Atmospheric temperature: $T_a = 30\,°C$
Temperature at the left face of wall: $T_1 = 75.0\,°C$.

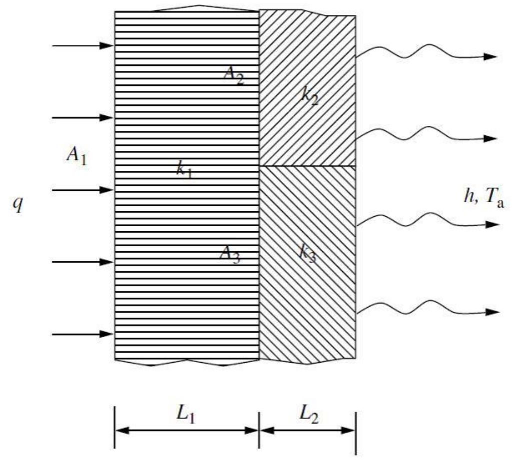


Fig. 3

Q.5B Deduce all shape functions for a Isoparametric triangular area element having primary and mid side nodes, using serendipity approach. (04)