Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
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A Constituent Institution of Manipal University

V SEM B.TECH. (BME) DEGREE MAKE UP EXAMINATIONS, DECEMBER 2017
SUBJECT: DIGITAL SIGNAL PROCESSING (BME 3104)
(REVISED CREDIT SYSTEM)
Friday, 29th December 2017, 2 PM to 5 PM

TIME: 3 HOURS **MAX. MARKS: 100**

Instructions to Candidates:

1. **Answer ALL questions.**
2. **Draw labeled diagram wherever necessary**

Consider the following

DFT	Discrete Fourier Transform	LTI	Linear Time Invariant
DTFT	Discrete Time Fourier Transform	FIR	Finite Impulse Response
ROC	Region Of Convergence	IIR	Infinite Impulse Response
BIBO	Bounded Input Bounded Output		

1 (a) Derive the necessary and sufficient conditions for a relaxed LTI system to be Causal. **[5]**

(b) Determine the DTFT of a rectangular Pulse defined as: **[5]**

$$x(n) = \begin{cases} 1, & |n| \le 2 \\ 0, & |n| > 2 \end{cases}$$

Plot the magnitude spectrum, with clear labelling of the specifications.

(c) Determine the transfer function of the simple 2nd order IIR Band Stop Filter for a given notch frequency $\omega_0 = \frac{\pi}{2}\ rad/sample$ and Bandwidth $B = \frac{\pi}{2}\ rad/sample$. Plot the magnitude and phase of the frequency response, with clear labelling of the specifications. **[10]**

2 (a) Determine the even and odd sequences associated with the finite length real sequence *x(n)* of length-7: **[5]**

$$x(n) : \{-1,\ \ -4,\ \ 2,\ \ -2,\ \ \underset{\uparrow}{1},\ \ 0,\ \ -2\},$$

(b) Consider the three sequences generated by uniform sampling the three cosine functions of frequencies 3 Hz, 7 Hz and 13 Hz respectively: $g_1(t) = \cos(6\pi t)$, $g_1(t) = \cos(14\pi t)$, and $g_1(t) = \cos(26\pi t)$ with a sampling rate of 10 Hz. Represent the sequences in discrete form. Are these discrete sequences represent the same? Explain? **[5]**

(c) Consider the following linear constant coefficient difference equation: **[10]**

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 2x(n-1)$$

Determine $y(n)$ using total solution method (direct method), when $x(n) = \delta(n)$ and $y(n) = 0, n < 0$.

3. (a) Derive the sufficient conditions for the existence of DTFT $X(e^{j\omega})$ of a sequence $x(n)$. **[5]**

(b) Consider a LTI system with impulse response **[5]**

$$h(n) = u(n) - u(n-5)$$

if the input to this system is $x(n) = a^n u(n)$, then determine the response of the system.

(c) Consider two 4-point real sequences given as: **[10]**

$$x(n) = \begin{cases} (-1)^n + n, & 0 \le n \le 3 \\ 0, & otherwise \end{cases}$$

$$h(n) = \begin{cases} 2 - n, & 0 \le n \le 3 \\ 0, & otherwise \end{cases}$$

Determine the Circular convolution using the convolution theorem of DFT. Use matrix method to compute the DFT and IDFT.

4 (a) State and prove the convolution theorem of DTFT. **[5]**

(b) Consider a signal that is the sum of two real exponentials: **[5]**

$$x(n) = \left(\frac{1}{3}\right)^n u(n) + (-5)^n u(n)$$

Determine the z-transform $X(z)$ of $x(n)$. Identify the ROC in the pole-zero plot of $X(z)$? Justify whether or not there exist Fourier Transform for $x(n)$ from pole-zero plot?

(c) Explain the Divide-and-Conquer approach to compute DFT of an N-point sequence $x(n)$. **[10]**

5 (a) Consider the system **[5]**

$$y(n) = nx(n^2)$$

Determine whether the system is Linear and Time Invariant.

(b) Consider an N-point sequence $g(n)$ whose N-point DFT is $G(k)$. Then derive the N-point DFT of $g(n)$ with a clockwise circular shift in time index by n_0. **[5]**

(c) Determine the Discrete Time sequence $x(n)$ if z-transform $X(z)$ is given by **[10]**

$$X(z) = \frac{1}{(1-0.5z^{-1})^2(1+0.6z^{-1})(1-0.2z^{-1})}, \qquad ROC: |z| > 0.6$$

using the Partial Fraction Expansion Method. Sketch the pole-zero plot.