Reg. No. | | | | | | | | | |

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
A Constituent Institution of Manipal University

VII SEMESTER B.TECH (ELECTRICAL & ELECTRONICS ENGINEERING)

END SEMESTER EXAMINATIONS, NOVEMBER 2017

SUBJECT: LINEAR ALGEBRA AND ITS APPLICATIONS TO ELECTRICAL ENGINEERING [ELE 4030]

REVISED CREDIT SYSTEM

Time: 3 Hours **Date: 21 November 2017** **Max. Marks: 50**

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1A. Define norm of a vector with relevant axioms. Find L_0, L_1, and L_∞ norms of the following vector/matrix.

$$A = (i,\ 2,\ 1-i,\ 0,\ 1+i)^T;\ B = \begin{pmatrix} 1 & 4 & 0 \\ 4 & 12 & 4 \\ 0 & 4 & 0 \end{pmatrix}.$$

(03)

1B. Consider the vectors $u = (2,\ 1,\ -4,\ -2)^T$ and $v = (1,\ -1,\ 1,\ -1)^T$.

(i) Determine the Euclidean distance between u and v.

(ii) Verify that the triangle inequality holds for u and v. ***(02)***

1C. If $x, y \in \mathcal{R}^n$, prove or disprove $||x-y|| = ||y-x||$ is true for all norms. ***(03)***

1D. Given the linear system $Ax = b$.

What are the three possibilities for the set of solutions? What is the condition for the system to be consistent? ***(02)***

2A. Reduce the following system to *RREF* (row reduced echelon form) using Gaussian elimination with backward/forward substitution.

$$v - w = 3$$

$$-2u + 4v - w = 1$$

$$-2u + 5v - 4w = -2$$

Find the solution using *RREF* if exists. Also mention whether the system is consistent. ***(04)***

2B. Using Gauss-Jordan technique find the inverse of the following matrix:

$$P = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

(03)

2C. True or false? Give a counterexample if false and a reason if true.

(i) In $Ax = b$ formulation, set of all possible x represent a vector space.

(ii) Number of basis vectors represents the dimension of the vector space.

(iii) If $Ax = 0$ for $x \neq 0$, then A has no inverse. ***(03)***

3A. Describe the column space of A, null space of A, and the complete solution to $Ax = b$. Given:

$$A = \begin{pmatrix} 2 & 4 & 6 & 4 \\ 1 & 2 & 3 & 6 \\ 0 & 0 & 1 & 2 \end{pmatrix}; \text{ and } b = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}.$$

(04)

3B. What is rank of a matrix? Explain the rank-nullity theorem. ***(02)***

3C. For which numbers c and d do these matrices have rank 2?

$$B = \begin{pmatrix} c & d \\ d & c \end{pmatrix}.$$

(02)

3D. State and explain any four properties of determinants. ***(02)***

4A. Derive expressions for determining unknown parameters of a circle to be fitted over sparsely spaced data points using least squares-based Kasa's circle-fitting technique. ***(04)***

4B. Find q_1, $q_{2,}$, and q_3 (orthogonal vectors) as combinations of a, b and c (independent vectors) in G.

$$G = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{pmatrix}$$

(04)

4C. Prove that the orthogonal matrices preserve length of a vector and inner product between vectors. ***(02)***

5A. Is 5 an eigenvalue of $A = \begin{pmatrix} 6 & -3 & 1 \\ 3 & 0 & 5 \\ 2 & 2 & 6 \end{pmatrix}$? ***(02)***

5B. Find the algebraic and geometric multiplicities of A. Diagonalize the following matrix A, if possible. $A = \begin{pmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{pmatrix}$

(04)

5C. Prove or disprove that the symmetric matrices have real eigenvalues and orthogonal eigenvectors. ***(02)***

5D. What are positive definite and semi-definite matrices? Mention few advantages. ***(02)***