



# MANIPAL INSTITUTE OF TECHNOLOGY MANIPAL

(A constituent Institution of MAHE, Manipal)

## VII SEMESTER B.TECH (ELECTRICAL & ELECTRONICS ENGINEERING) MAKEUP EXAMINATIONS, DECEMBER 2017

### SUBJECT: LINEAR ALGEBRA AND ITS APPLICATIONS TO ELECTRICAL ENGINEERING [ELE 4030]

REVISED CREDIT SYSTEM

Time: 3 Hours

Date: 26 December 2017

Max. Marks: 50

#### Instructions to Candidates:

- ❖ Answer **ALL** the questions.
- ❖ Missing data may be suitably assumed.

- 1A. Define norm of a matrix with relevant axioms. Find  $L_0$ ,  $L_1$ ,  $L_{Frobenious}$ , and  $L_\infty$  norms of the following matrix.

$$B = \begin{pmatrix} 1 & 6 & 0 \\ 3 & 10 & 4 \\ 0 & 5 & 2 \end{pmatrix}$$

(03)

- 1B. State and prove the Cauchy-Schwarz and triangular inequality for vectors  $x, y \in \mathcal{R}^2$

(04)

- 1C. Given the linear system  $Ax = b$ .

What are under defined, over defined, and well defined systems? Give their RREF form after elimination.

(03)

- 2A. Reduce the following system to RREF (row reduced echelon form) using Gaussian elimination with backward/forward substitution.

$$u + 2v + w = 2$$

$$2u + 4v = 2$$

$$3u + 6v + w = 4$$

Find the solution using RREF if exists. Also mention whether the system is consistent.

(04)

- 2B. Using Gauss-Jordan technique find the inverse of the following matrix:

$$P = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 5 & 7 \\ 3 & 6 & 8 \end{pmatrix}$$

(03)

- 2C. True or false? Give a counterexample if false and a reason if true.

- (i) In  $Ax = 0$  formulation, set of all possible  $x$  represent a vector space.
- (ii) Rank of a matrix is the dimension of the column space.
- (iii) Product of pivots = determinant of a matrix

(03)

- 3A. Describe the column space and null space of  $A$  and the complete solution to  $Ax = b$ . Given:

$$A = \begin{pmatrix} 2 & 4 & 6 & 4 \\ 1 & 2 & 3 & 6 \\ 0 & 0 & 1 & 2 \end{pmatrix}; \text{ and } b = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}.$$

(04)

**3B.** Prove or disprove that reflection matrices are orthogonal matrices. **(02)**

**3C.** Determine the basis and rank of the matrix given below?

$$B = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 4 & 2 & 2 \\ 3 & 6 & 3 & 4 \end{pmatrix}. \quad \mathbf{(02)}$$

**3D.** Prove any two properties of determinants. **(02)**

**4A.** Derive expressions for determining unknown parameters of a line to be fitted over sparsely spaced data points using least squares technique. **(04)**

**4B.** Find  $q_1$ ,  $q_2$ , and  $q_3$  (orthogonal vectors) as combinations of  $a$ ,  $b$  and  $c$  (independent vectors) in  $G$ .

$$G = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & -3 \\ 0 & -2 & 3 \end{pmatrix} \quad \mathbf{(04)}$$

**4C.** State and prove the Cayley-Hamilton theorem applicable to eigenvalues. **(02)**

**5A.** What are the eigenvalues and eigenvectors of  $A = \begin{pmatrix} 6 & -3 & 1 \\ 3 & 0 & 5 \\ 2 & 2 & 6 \end{pmatrix}$ ? **(04)**

**5B.** What are Hermitian, positive definite, unitary, and symmetric matrices? **(04)**

**5C.** Prove or disprove that the symmetric matrices have real eigenvalues and orthogonal eigenvectors. **(02)**