Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
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SEVENTH SEMESTER B.Tech. (E & C) DEGREE END SEMESTER EXAMINATION
NOV 2017
SUBJECT: INFORMATION THEORY AND CODING (ECE -407)

TIME: 3 HOURS **MAX. MARKS: 50**

Instructions to candidates
- Answer **ANY FIVE** questions.
- Missing data may be suitably assumed.

1A. Let S be zero memory source with source alphabet $S = \{s_i\}, i = 1,2, \dots, q$, and symbol probabilities $P_1, P_2, \dots, P_q$. Construct a new zero memory source S' with twice as many symbols, $S' = \{s_i'\}, i = 1,2, \dots ,2q$. Let P_i', the symbol probabilities for the new source, defined by

$$P_i' = (1 - \epsilon)P_i, \quad i = 1,2, \dots, q$$

$$P_i' = \epsilon P_{i-q} \qquad i = q + 1, q + 2, \dots ,2q.$$

Express $H(S')$ in terms of $H(S)$.

1B. One digit out of nine (1 to 9 inclusive) is chosen at random. Describe the sample space. and find the probability that the chosen number is (i) even (ii) odd (iii) prime.

1C. Define Entropy. Find the Entropy of a source S in bits, nats and heartleys which has the probability distribution P = {1/4,1/4,1/8,1/8,1/8,1/16,1/16}.

(5+3+2)

2A. Explain the properties of entropy.

2B. Starting from the logarithmic inequality, show that the entropy of the discrete memoryless source is maximum if its symbols are equi-probable.

2C. A binary source emitting an independent sequence of 0s and 1s with probabilities p and (1-p) respectively. Plot the entropy of this source versus p ($0<p<1$).

(5+3+2)

3A. Construct the Huffman and Shannon Fano Binary codes for the source emitting six symbols with probabilities 1/3, 1/3,1/6, 1/6, 1/12, 1/24, 1/24. Compute the code efficiency and redundancy for both codes. Which code is a compact code?

3B. Prove that for a DMS, $H(S^n)=nH(S)$.

3C. Give an alternative expression for H(Y) - H(Y /X) in terms of the joint entropy and both marginal entropies

(5+3+2)

4A. Given the table:

S	S_1	S_2	S_3	S_4	S_5	S_6	S_7
$F(S_i)$	1/3	1/3	1/9	1/9	1/27	1/27	1/27

a) Find H(s), $H_3(s)$
b) Find compact code for H(s) when X={0, 1} and X={0, 1, 2}
c) Compute L for the above codes.

4B. Which of the sets of word lengths shown below are acceptable for uniquely decodable code when the code alphabet is X={0,1,2}

Code A with 11 symbols of word lengths 1,2,2,2,2,3,3,3,3,3,3

Code B with 11 symbols of word lengths 1,1,2,2,3,3,4,4,5,5,5

Construct an instantaneous code if lengths are acceptable.

4C. An experiment consists in throwing first a die and then a coin. Describe a sample space of the experiment.

(5+3+2)

5A. Encode **HUFFMAN** using adaptive Huffman coding procedure.

5B. Define the channel capacity. Find the channel capacity of a channel whose matrix is given by

$$P(B/A) = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.5 & 0.3 & 0.2 \\ 0.2 & 0.5 & 0.3 \end{bmatrix}$$

5C. Find the capacity of the channel whose matrix is $\begin{bmatrix} (1-p-q) & q & p \\ p & q & (1-p-q) \end{bmatrix}$

(5+3+2)

6A. Consider a third order binary Markov source where the probability of emitting a 0 does not depend upon previous two symbols but does depend upon the third symbol back. The probability that the next symbol will be same as the third symbol back is 0.9, the probability that it will differ is 0.1. Sketch the state diagram for this source. Compute state probabilities. Compute the entropy of this source as well as the entropy of its adjoint.

6B. Cascade the channel $P(B/A) = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{bmatrix}$ with the channel $P(C/B) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2/3 & 1/3 \\ 0 & 1/3 & 2/3 \end{bmatrix}$

Find I(A;B) and I(A;C) if input symbols are equiprobable.

6C. The binary symmetric channel has the probability of error 0.01. Show how to reduce this error to the order of 10^{-5}. What is the penalty for achieving this error?

(5+3+2)