Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

A Constituent Institute of Manipal University, Manipal

VII SEMESTER B.TECH
(MECHANICAL/INDUSTRIAL AND PRODUCTION ENGINEERING)
END SEMESTER MAKEUP EXAMINATION – DEC. 2017
SUBJECT: FINITE ELEMENT METHODS (MME 4102)
REVISED CREDIT SYSTEM

Time: 3 Hour **Max. Marks: 50**

Note: (i) Missing data, if any, may be appropriately assumed
(ii) Draw sketches as applicable
(iii) Assumptions made must be clearly mentioned

1A. The governing differential equation of a particular boundary value problem is given by $195\frac{d^2y}{dx^2}+105\frac{dy}{dx}=60$. It has a x bound of $0 \le x \le 7$. Its boundary conditions are $y(x = 0) = 0$ and $y(x = 7) = 0$. Use Galerkin's method to find an approximate solution to this differential equation. Consider a suitable polynomial approximation function for beam deflection. 05

1B. Find the integrals using (i) 1 point Gauss-Quadrature rule, and (ii) 2 point Gauss-Quadrature rule, 03

(a) $I=\int_{-1}^{+1}\left(\frac{\cos x}{1-x^2}\right)dx$ (b) $K=\int_{-1}^{+1} xcox\,dx$

1C. Explain the Half band width method and the Skyline method of storing the large sized banded matrices in computers. 02

2A. Consider an irregular quadrilateral plate ABCD with corners coordinates A(15,70), B(50,30), C(110,40) and D(70,60). Use 2 triangular elements (ABD and BCD) to model the quadrilateral plate. Using shape functions expressed in area coordinates, 05

(i) Find the shape functions at A, B, C and D as functions of (x, y) with respect to both the elements ABD and BCD.

(ii) Find the x direction displacement component at a point P(55, 50) within the element ABD if the x direction displacement components at A, B and D are respectively +2 mm, +5 mm and -2.5 mm.

(iii) Find the y direction displacement component at a point Q(80, 45) within the element BCD if the y direction displacement components at B, C and D are respectively –3 mm, +1 mm and +2.25 mm.

2B. Derive the shape functions of a higher order one dimensional element consisting of 6 nodes using the Lagrange polynomial function. 03

2C. With reference to FEM, define the term '*Convergence*' and explain any one method of attaining convergence. 02

3A. Use FEM to determine (a) the fluid head distribution along the length of the coarse gravelly medium shown in Figure Q (3A), (b) the velocity in the upper part, and (c) the volumetric flow rate in the upper part. The fluid head at the top is 500 mm. and that at the bottom is 20 mm. Assume a permeability coefficient K = 15 mm/s and cross-sectional area A = 800 mm^2. 05

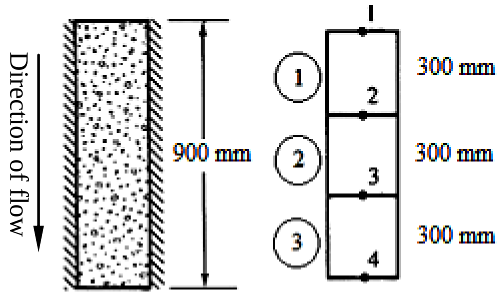

Fig. Q (3A)

3B. The discretized structure shown in Figure Q (3B) consists of 3 elements. A traction force of 0.3 kN/mm is acting on element (2). Point loads of 20 kN, 10 kN and 8 kN are acting on the structure as shown in figure. Using FEM determine the (i) Global stiffness matrix and (ii) Global force vector. Use minimum number of elements. 03

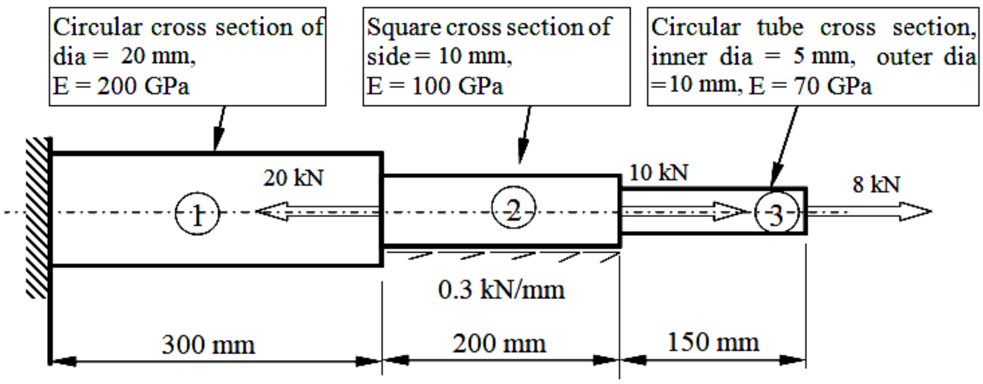

Fig. Q (3B)

3C. Derive the expression for element stiffness matrix for 2D Truss. 02

4A. For the CST element shown in Fig. Q (4A), the coordinates of the nodes are given in units of millimeters. Assume plane stress conditions, Young's modulus E = 210 GPa, Poisson's Ratio ν = 0.25, and thickness t = 10 mm. If the nodal displacements are given as: At node 1, (q_1 = 2.0 mm, q_2 = 1.0 mm), at node 2, (q_3 = 0.5 mm, q_4 = 0.0 mm) and at node 3 (q_5 = 3.0 mm and q_6 = 1.0 mm), use FEM to determine 05

(i) the Jacobian matrix

(ii) the strain displacement matrix

(iii) the element strain components: ε_{xx}, ε_{yy} and γ_{xy}

(iv) the element stress components: σ_{xx}, σ_{yy} and τ_{xy}

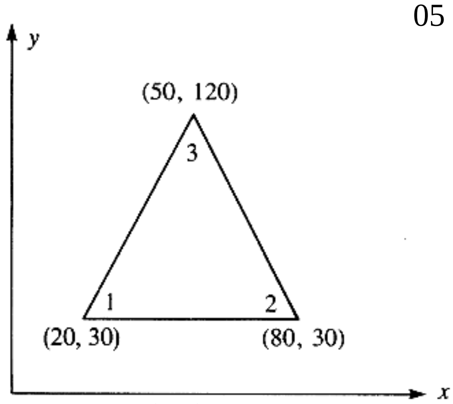

Fig. Q (4A)

4B. Consider the plane truss structure shown in Fig. Q (4B). The structure is made of three planar truss members as shown, and a vertical downward force of 1000 N is applied at node 2. The figure shows the numbering of the elements (labelled in squares), as well as the numbering of the nodes (labelled in circles). For each member take area of cross section = 500 mm^2 and Young's modulus = 70 GPa. If the support at node 3 has a leftward settlement of 2 mm, use FEM to 03

(i) Assemble the global stiffness matrix.

(ii) Apply Elimination approach of handling boundary conditions and write the reduced FEM matrix relationship.

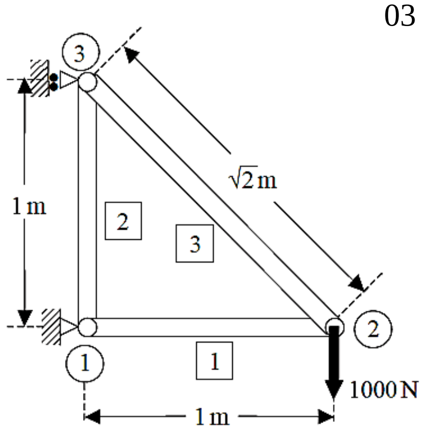

Fig. Q (4B)

4C. Derive the expression for converting the uniformly distributed load acting on a beam element into its equivalent forces and moments to be applied at the nodes. 02

5A. The beam structure shown in Fig. Q (5A) is subjected to a triangular distributed load and a point load. Use only two elements, one for the portion where the triangular distributed load is acting and the second one for the remaining part of the beam. Use FEM and find the deflections and slopes at the nodes. 04

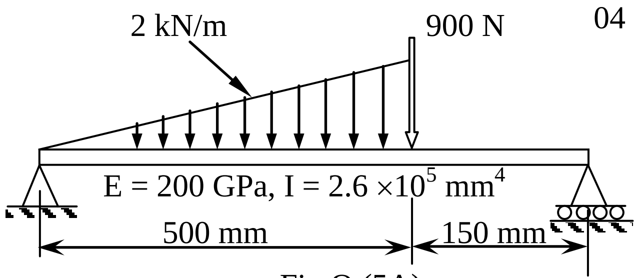

Fig Q (5A)

5B. Derive the expression for Element traction force vector of a 1D bar element using shape functions. 02

5C. Compute the mass matrices for different element types given below. For each case take density ρ = 7850 kg/m^3, area of cross section A = 300 mm^2, length L = 200 mm. 04

(i) Consistent mass matrix for a one dimensional element.

(ii) Consistent mass matrix for a planar truss element.

(iii) Lumped mass matrix for a planar truss element.

(iv) Lumped mass matrix for a beam element.

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