Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

A Constituent Institute of Manipal University, Manipal

VII SEMESTER B.TECH (MECHANICAL ENGINEERING / INDUSTRIAL AND PRODUCTION ENGINEERING)

END SEMESTER EXAMINATION – NOV 2017

SUBJECT: FINITE ELEMENT METHODS (MME 4102)

REVISED CREDIT SYSTEM

Time: 3 Hours **Max. Marks: 50**

Note: (i) Missing data, if any, may be appropriately assumed
(ii) Draw sketches as applicable
(iii) Assumptions made must be clearly mentioned

1A. The potential energy of a cantilever beam subjected to point force at its free end is given by

$$\Pi = \frac{EI}{2}\int_0^L \left(\frac{d^2y}{dx^2}\right)^2 dx - P\,y_{(x=L)}$$

where, y is the deflection of the beam, EI is the flexural rigidity and P is the point load.

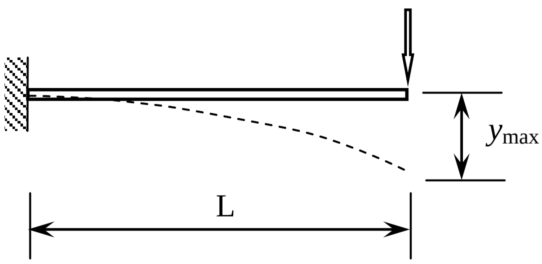

Fig. Q1A

(i) Determine the maximum deflection (y_{max}) of a cantilever beam (shown in Fig. Q1A) which is subjected to a point load P at its free end. Use Rayleigh Ritz method assuming a cubic polynomial function.

(ii) Based on the relationship of maximum deflection derived, calculate the y_{max} for L = 3m, P = 2 kN, for a section having flexural rigidity $EI = 2.67 \times 10^{10}$ N mm^2. 05

1B. Use Gaussian quadrature (2 × 2) rule to evaluate the double integrals:

(i) $\int_0^1\int_0^2 xy\,dx\,dy$ (ii) $\int_{-1}^{+1}\int_{-1}^{+1}\left(r^2+2rs+s^2\right)dr\,ds$ 03

1C. List the general steps of FEM. 02

2A. Consider a triangular element with nodes 1, 2 and 3 with their (x, y) coordinates given as (20, 10), (80, 30) and (50, 70) respectively. A point P with (x, y) coordinates (50, 40) is located inside the triangular element.

(i) Use shape functions to determine the local coordinates (ζ, η) of point P.

(ii) Draw the element to scale and show the location of point P.

(iii) Determine the values of the shape functions at the point P.

(iv) The x and y displacement components at the nodes are given in Table Q 2A. Use shape functions to determine the x and y displacement components (u and v) at the point P.

Table Q 2A

Node #	**Nodal displacements**	
	***x* component**	***y* component**
1	2 mm (acting to the right)	1 mm (acting downwards)
2	3 mm (acting to the left)	2 mm (acting upwards)
3	1 mm (acting to the right)	2 mm (acting upwards)

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2B. Derive the shape functions of a 4 noded quadrilateral element. 03

2C. With an example explain the reasons for using polynomials as displacement functions in FEM. 02

3A. For the discretized 1D aluminium bar shown in Fig. Q 3A the temperature at node 1 is given to be 200°C. The ambient temperature is 30°C. The thermal conductivity (K) of aluminium is 210 W/m °C. The convective heat transfer coefficient h is 30 W/m^2 °C. Consider the heat transfer due to conduction, convection from the peripheral surface and free end convection from node 4.

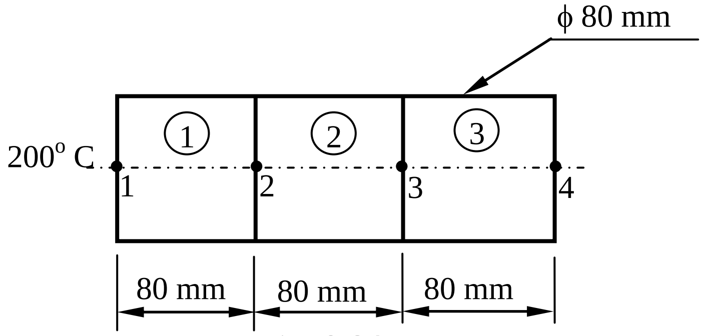

Fig. Q 3A

Neglect the free end convection from the left end at node 1.
Determine the

(i) temperatures at the nodes 2 through 4, and

(ii) rate of heat flow in the 3 elements.

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3B. Consider the bar shown in Fig. Q 3B. There is an increase in temperature of 60°C for both the materials. A point load of 1000N is acting at the interface of the two materials, as shown in the Fig. Use the data given in Table Q 3B. Apply FEM to determine:

(i) Global stiffness matrix

(ii) Global force vector

(**NOTE: DO NOT SOLVE)**

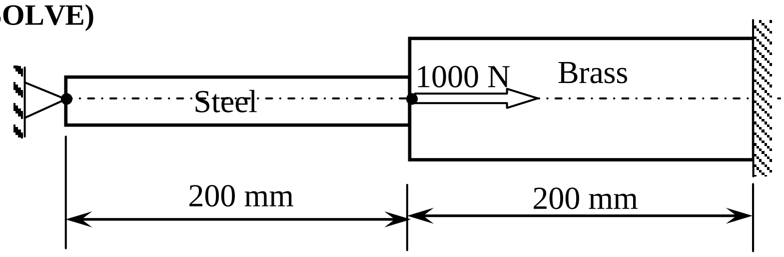

Fig. Q 3B

Table Q 3B						
Material	Area of cross section (mm^2)	Young's Modulus (GPa)	Coefficient of thermal expansion ($\times 10^{-6}/^oC$)	Body force (N/mm^3)	Traction force (N/mm)	Change in temperature (^oC)
Steel	200	200	12	Nil	5	60
Brass	400	105	19	0.01	Nil	60

03

3C. Derive the expression for the thermal force vector for a 2D truss element. 02

4A. Fig. Q 4A shows a Constant Strain Triangular element subjected to point loads (at nodes 2 and 3), traction load (on edge 1-2) and body force (self-weight). The material of the element is steel with Young's Modulus, E = 200 GPa, Poisson's ratio, $\nu = 0.3$, density, $\rho = 7850$ kg/m^3. The thickness of the element (perpendicular to the plane of paper) is t = 3 mm.

(i) Assemble the global force vector.

(ii) Determine the strain-displacement matrix.

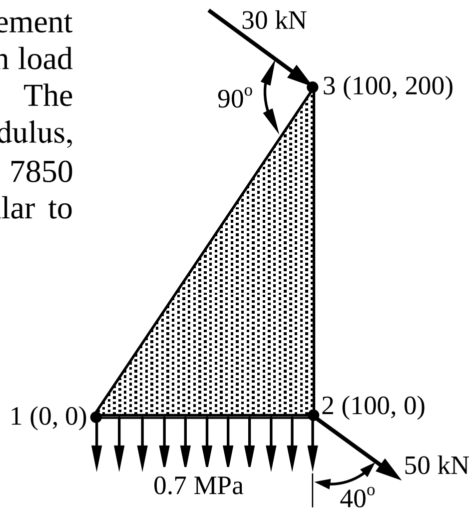

Fig. Q 4A

All dimensions in mm

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4B. For the two-bar truss shown in Fig. Q 4B, determine the displacement of node 1. A horizontal force of P = 1000 kN (acting towards left) is applied at node 1. Node 1 settles by an amount δ = 50 mm (downward) in the vertical direction. Take the Young's modulus, E = 210 GPa and Area of cross section, A = 600 mm^2 for each element.

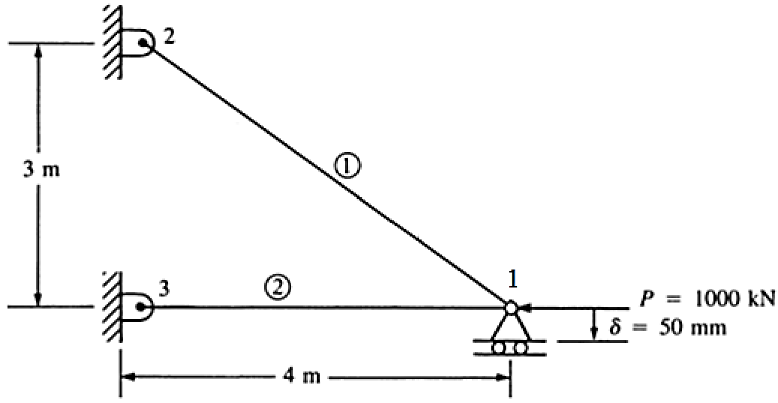

Fig. Q 4B

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4C. Write the Hermite shape functions for a beam element that are used to represent displacement. Also plot their variations over the length of the beam element. 02

5A. A discretized beam structure shown in Fig. Q 5A has a constant diameter of 20 mm throughout its length. It is subjected to trapezoidal and triangular distributed loads. Use FEM to model the problem.

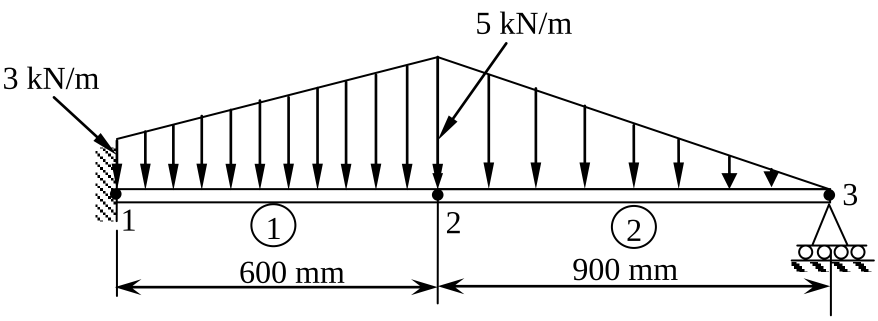

Fig. Q 5A

Assume Young's modulus, E = 200 GPa

(a) Assemble the global stiffness matrix
(b) Assemble the global force vector
(c) Use Elimination approach to handle the boundary conditions and write the resulting reduced matrix relationship. (**NOTE: DO NOT SOLVE)** 03

5B. Derive the expression for Element body force vector for a 1D bar element using shape functions. 02

5C. Consider the axial vibration (along the direction x) of the stepped steel bar shown in Fig. Q 5C.

(a) Assemble the global mass matrix and the global stiffness matrix
(b) Determine the natural frequencies of the rod using the characteristic polynomial method.

Consider one element per section of the rod. Assume Young's modulus E = 200 GPa, density, ρ = 7850 kg/m^3

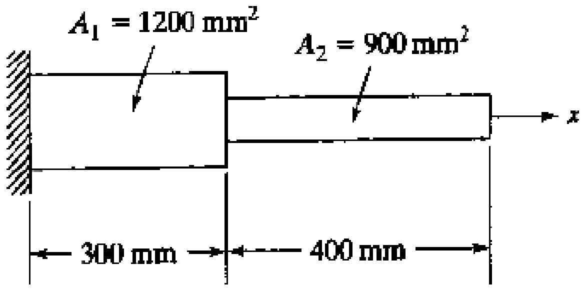

Fig. Q 5C

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