Question Paper

Exam Date & Time: 03-May-2018 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END-SEMESTER THEORY EXAMINATION- MAY 2018
I SEMESTER B.S.(ENGINEERING)
DATE:03.05.2018
TIME:09:30AM TO 12.30PM

Mathematics - I [MA 111]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.

1)

A) Find the n^{th} derivatives of i) $e^{ax}\sin(bx+c)$. ii) x^3e^{ax}. (8)

B) Evaluate i) $\int_0^{\pi}\frac{\sqrt{1-\cos x}}{1+\cos x}\sin^2 x\,dx$ ii) $\int_0^{\infty}\frac{dx}{(a^2+x^2)^n}$. (8)

C) Trace the curve $xy^2 = a^2(a-x)$ with explanations. (4)

2)

A) If $0 < a < b$ then, prove that $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}$ and hence deduce that $\frac{\pi}{4}+\frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4}+\frac{1}{6}$. State the result used. (8)

B) Trace the curve $r = a(1+cos\theta)$ with explanations. (8)

C) If $y = e^{a\sin^{-1}x}$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. (4)

3)

A) Prove that the radius of curvature at any point P of the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is three times the length of the perpendicular from the origin onto the tangent to the curve at P. (8)

B) Trace the curve $(x^2+y^2)x - a(x^2-y^2) = 0$ with explanations. (8)

C) Find the radius of curvature of the curve $x^2y = a(x^2+y^2)$ at the point $(-2a, 2a)$. (4)

4)

A) Find the area of the portion included between $r = a(1+cos\theta)$ & $r = a\ (1-cos\theta)$. (8)

B) Show that the curves $r^2 = a^2cos2\theta$ & $r = a\ (1+cos\theta)$ intersect at an angle of $3\sin^{-1}\left(\frac{3}{4}\right)^{\frac{1}{4}}$ (8)

sin (4) .

C) A variable plane at constant distance 3p from the origin meets the axes in A, B, C. Show that the locus of the centroid of the Δ ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$. (4)

5) A) Show that evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $(ax)^{\frac{2}{3}}+(by)^{\frac{2}{3}}=(a^2-b^2)^{\frac{2}{3}}$. (8)

B) Find the length of the arc of the parabola $y^2=4ax$ measured from the vertex to one extremity of the latus rectum. (8)

C) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}x^{2n}$. (4)

6) A) Find the image of the point (1, −2, 3) in the plane $2x-3y+2z+3=0$. (8)

B) State the values of x for which the following series converge (8)

$$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\cdots\infty.$$

C) Find the equation of the plane through the origin and containing the line (4)

$$\frac{x-1}{5}=\frac{y-2}{4}=\frac{z-3}{5}.$$

7) A) Expand $\log(1+\sin x)$ in powers of x by Maclaurin's theorem up to terms containing x^4. (8)

B) Test the convergence of the series $\frac{x}{1+x}+\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}+\cdots\infty$. (8)

C) Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0$, $x+y+z=3$ as a great circle. (4)

8) A) Find the magnitude and the equation of the line of shortest distance between the lines (8)

$$\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}\ \&\ \frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}.$$

B) Evaluate $\lim_{x\to o}\frac{(1+x)^{\frac{1}{x}}-e}{x}$. (8)

C) (4)

Find the equation of the plane passing through the point $(1,1,3)$ and parallel to the plane

$$3x+4y-5z=0 .$$

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