Question Paper

Exam Date & Time: 03-May-2018 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END-SEMESTER THEORY EXAMINATION- MAY 2018
I SEMESTER B.Sc.(Applied Sciences)
DATE:03.05.2018
TIME:09.30AM TO 12.30PM
MATHEMATICS - 1 [IMA 111]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.

1) A) Find the n^{th} derivatives of i) $e^{ax}\sin(bx+c)$. ii) x^3e^{ax}. (8)

B) Obtain a reduction formula for $\int \cos^m x \sin^n x dx$ when m and n are non-negative integers. Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^m x \sin^n x dx$. (8)

C) Trace the curve $x = a\cos^3\theta, y = b sin^3\theta, a > b$ with explanations. (4)

2) A) If $y = e^{m\cos^{-1}x}$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. (8)

B) Integrate the following: (i) $\int_0^{2a} x^2\sqrt{2ax-x^2}dx$ (ii) $\int_0^{\infty} \frac{1}{(1+x^2)^{\frac{7}{2}}}dx$ (8)

C) Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$ (4)

3) A) Prove that the radius of curvature at any point P of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is three times the length of the perpendicular from the origin onto the tangent to the curve at P. (8)

B) Find the co-ordinates of curvature at any point of the parabola $y^2 = 4ax$ Hence show that its evolute is $27ay^2 = 4(x-2a)^3$ (8)

C) Find the angle of intersection of the curves $r = (sin\theta + cos\theta)$ and r=2sinθ. (4)

4) A) Find the volume of the solid generated by the curve $xy^2 = 4a^2(2a-x)$ about y-axis. (8)

B) Find the image (reflection) of the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{4}$ in the plane $2x+y+z=-2$. (8)

C) Find the area of the loop of the curve $3ay^2=x(x-a)^2$. (4)

5) (8)

A) Find the surface of the solid generated by the revolution of the leminscate $r^2=a^2\cos 2\theta$

B) Test the convergence of the series $\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\cdots\infty$. (8)

C) Obtain the perimeter of the cardiod r=a(1+cosθ). (4)

6) (8)

A) Find the equation of the right circular cylinder having the circle $x^2+y^2+z^2=9,\ x-y+z=3$ as base circle.

B) (i) Find 'c' such that $f(x)=\sqrt{x}$ and $g(x)=\frac{1}{\sqrt{x}}$ satisy Cauchy's mean value theorem in $\left[\frac{1}{4},1\right]$ (8)

(ii) Evaluate $\lim_{x\to 0}\frac{x\sin x}{(e^x-1)^2}$

C) Find the point where the line $\frac{x-2}{2}=\frac{y-4}{-3}=\frac{z+6}{4}$ meets the plane $2x+4y-z-2=0$ (4)

7) (8)

A) Find the equation of the cone whose vertical angle is $\frac{\pi}{2}$, which has its vertex at the origin and its axis along the line x=-2y=z

B) Evaluate $\lim_{x\to\infty}\left(\frac{ax+1}{ax-1}\right)^x$ (8)

C) Test the convergence of the series $\sum_{n=1}^{\infty} ne^{-n^2}$ and mention the test used. (4)

8) (8)

A) Expand $\tan^{-1}x$ in powers of $(x-1)$ up to third degree terms by Taylor's theorem.

B) (8)

(i) Test for conditional convergence of the series $\sum\frac{(-1)^{n-1}n}{n^2+1}$

(ii) Write the Macluarian series expansion of $\sqrt{1-x}$ upto third degree term.

C) Find the tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$ which are parallel to the plane 2x+2y-z=0. (4)

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