Question Paper

Exam Date & Time: 31-May-2018 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
II SEMESTER B.Sc.(Applied Sciences) DEGREE MAKE UP- EXAMINATION - MAY / JUNE 2018
DATE: 31 MAY 2018
TIME : 9.30 AM TO 12.30 PM
Mathematics - II [IMA 121]

Marks: 100 **Duration: 180 mins.**

Answer ANY FIVE full Questions.
Missing data, if any, may be suitably assumed

1)

A) If $r^2 = x^2 + y^2 + z^2$ and $v = r^m$ then, prove that (7)

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = m(m+1)\, r^{m-2}$$

B) Find the minimum value of $x^2 + y^2 + z^2$, given that $ax + by + cz = p$. (7)

C) Show that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin\theta}}\, d\theta = \pi$ (6)

2)

A) Find by double integration, the area enclosed by the curves $x^2 + y^2 = a^2$ and $x + y = a$ (7)

B) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, prove that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$ (7)

C) Test for Consistence and solve the following equations by Gauss elimination method, $2x + 3y + z = 6$, $x + 2y + z = 4$, $4x - 3y + z = 2$. (6)

3)

A) (7)

State Stokes' theorem. Verify Stokes's theorem for $F = x^2 i + xyj$ around the square $x=0, x=a, y=0,$ $y=a$ in the plane $z=0$

B) (7)

Define rank of a matrix. Find the rank of

$$\text{matrix } A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}.$$

C) (6)

If $u = \text{Sin}^{-1}\left[\dfrac{x^3 - y^3}{x - y}\right]$ then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2\tan u.$

4) (7)

A)

Using elementary row operations, find the inverse of

$$\text{the matrix } A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}.$$

B) (7)

Define orthogonal set of vectors and orthonormal vectors. Using Gram Schmidt process construct an orthonormal set of basis vectors for the set of vectors $\{(1,1,1), (0,1,1), (1,1,0)\}$

C) (6)

If the kinetic energy T is given by $T = \dfrac{1}{2} mv^2$ find approximately the change in T as m changes from 49 to 49.5 and v changes from 1600 to 1590.

5) (7)

A)

Evaluate $\int^{\infty}\int^{\infty} e^{-y}$ dxdy by changing the order of integration

B) (7)

State Gauss divergence theorem. Evaluate $\iint_S F.\,nds$, where

$F= 4xzi - y^2 j + yzk$ and S is the surface of the cube

bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1,$

C) (6)

Prove that the set $A = \{(1,1,1),(1,0,1),(0,1,1)\}$ forms a basis for R^3 and express $(1,2,3)$ in terms of elements of A.

6) (7)

A) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$

B) (7)

State Green's theorem in the plane. Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\int_C (xdy - ydx)$. Find the area

C) (6)

Find by double Integration, the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

7) (7)

A) Show that the following system of equations are consistent and solve them. $x + y + z = 6,\ x + 2y + 3z = 14,\ x + 4y + 7z = 30.$

B) (7)

If $u = f(r)$, where $r^2 = x^2 + y^2$ prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

C) (6)

Find an equation for the tangent plane to the surface $x^2yz - 4xyz^2 + 6 = 0$ at the point (1, 2, 1).

8) (7)

A)

Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-x^2-y^2}\,dydx$, hence find $\int_0^{\infty} e^{-x^2}\,dx$

B) Verify the set {(1, 2, 1), (2, 4, 0), (2, -2, 0),(2, -2, 2)} of vectors is linearly dependent? (7)

C) Given $\phi = 6x^3y^2z$. Find div(grad ϕ) and curl(grad ϕ). (6)

-----End-----