# Question Paper

Exam Date & Time: 17-Apr-2018 (09:30 AM - 12:30 PM)

## MANIPAL ACADEMY OF HIGHER EDUCATION

**INTERNATIONAL CENTRE FOR APPLIED SCIENCES**
**SECOND SEMESTER B.S. (ENGG)**
**END-SEMESTER THEORY EXAMINATIONS APRIL - 2018**
**DATE: 17 APRIL 2018**
**TIME: 9:30AM TO 12:30PM**

**Mathematics - II [MA 121]**

**Marks: 100** **Duration: 180 mins.**

### Answer 5 out of 8 questions.
### Missing data, if any, may be suitably assumed

1) A) Find the maximum and minimum values of the function $f(x,y) = x^3 + y^3 + 3xy$ (7)

B) Change the order of integration and hence evaluate, $\int_0^1 \int_{x^2}^{2-x} xy \, dy\, dx$ (7)

C) Define a basis for $E^n$. Test whether the set B={(1,1,0), (3,0,1), (5,2,2)} forms a basis for $R^3$. If so, represent (1, 2, 3) in terms of basis vectors. (6)

2) A) Define an orthonormal basis of vectors and construct an orthonormal basis from the set of basis vectors {(0,1,0), (2,3,0), (0,2,4)} by using Gram Schmidt process. (7)

B) Evaluate $\iint_S^{\square} \vec{F}.\vec{n}\, dS$, where $\vec{F} = (x+y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and S is the surface of the plane 2x + y + 2z = 6. (7)

C) Find the area of the region bounded by the curves $y = 2 - x^2$ and $y = x$ (6)

3) A) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2). (7)

B) The temperature T at any point (x,y,z) in space is $400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$ , using Lagrange's method of (7) undetermined multipliers.

C) Show that the following system of equations are consistent and solve them. x + y + z = 6, x + 2y + 3z = 14, x + 4y + 7z = 30. (6)

4) Verify Stoke's theorem for $\vec{A} = y^2\vec{i} + xy\vec{j} - xz\vec{k}$ , where S is (7)

A) the upper half of the spher $x^2 + y^2 + z^2 = a^2, z \geq 0$ .

B) Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes y+z=4 and z=0 (7)

C) If u=f(2x-3y, 3y-4z, 4z-2x) , then prove that $6u_x+4u_y+3u_z=0$ (6)

5) (7)

A) Prove that $\nabla|\vec{r}|^n = n|\vec{r}|^{n-2}\vec{r}$. Hence evaluate $div\left(\frac{\vec{r}}{|\vec{r}|^3}\right)$.

B) Verify Green's theorem in the plane for $\phi(y - \sin x)dx + \cos x\, dy$. The closed curve of the region is the triangle bounded by the vertices (0,0), $(0,\frac{\pi}{2})$ and $(\frac{\pi}{2},1)$ (7)

C) Evaluate $\iint_R (x^2 + y^2)dx\, dy$, where R is the region bounded by y=x and $y^2=4x$. (6)

6) If $r^2=x^2+y^2+z^2$ and $V=r^m$ then, prove that (7)

A) $V_{xx}+V_{yy}+V_{zz}=m(m+1)r^{m-2}$

B) (7)

Find the rank of a matrix $\begin{pmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$ by reducing into Echelon form.

C) Prove that $\nabla \times (\emptyset\vec{A}) = (\nabla\emptyset) \times \vec{A} + \emptyset(\nabla \times \vec{A})$. (6)

7) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then prove that (7)

A) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$

B) Find the volume of the region enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the paraboloid $z=x^2+y^2$ (7)

C) Evaluate $\int_0^\infty xe^{-x^8}dx \times \int_0^\infty x^2e^{-x^4}dx$. (6)

8) (7)

A)

Find the inverse of a matrix $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}$ by using elementary transformations.

B) Is $\vec{F} = (y^2 cosx + z^3)\hat{\imath} + (2y\ sinx - 4)\hat{\jmath} + (3xz^2 + 2)\hat{k}$ is conservative ? If so find scalar potential. (7)

C) Prove that $\beta(p, q) = \frac{\Gamma(p)\ \Gamma(q)}{\Gamma(p+q)}$. (6)

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