Question Paper

Exam Date & Time: 02-May-2018 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
THIRD SEMESTER B.S (ENGG)
END-SEMESTER THEORY EXAMINATIONS APRIL - 2018
DATE : 2 MAY 2018
TIME : 9:30AM TO 12:30PM
MATHEMATICS - III [MA 231]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.
Missing data, if any, may be suitably assumed

1)

A) Solve: $\frac{dy}{dx} = (4x + y + 1)^2.$ (7)

B) Solve by the method of separation of variables (7)

$$x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} = 0.$$

C) Find the inverse Laplace transform of $\frac{s+3}{s^2-4s+13}.$ (6)

2)

A) Solve: $\frac{dy}{dx} + x \sin 2y = x^3 cos^2 y.$ (7)

B) Solve $\frac{dy}{dx} = 2y + 3e^x$ given $y(0) = 0$ and find y at x = 0.1 and x = 0.2 to five decimal places taking h = 0.1 by Taylor's series method (carry up to fourth order derivatives). (7)

C) Apply Convolution theorem to evaluate $L^{-1}\left[\frac{s^2}{(s^2+16)(s^2+9)}\right].$ (6)

3)

A) Solve: $(xy^3 + y)\, dx + 2(x^2y^2 + x + y^4)\, dy = 0.$ (7)

B) Solve $\frac{dy}{dx} = \log(x + y),\ y(0) = 2$, by Euler's modified method, for x = 0.2 and x = 0.4 taking h = 0.2. Carry out two iterations for each step. (7)

C) Solve $y'' + 5y' + 6y = 5e^{2t},\ y(0) = 2, y'(0) = 1$ using Laplace (6) transform method.

4) A) Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2x} sinx.$ (7)

B) Given $\frac{dy}{dx} = x + y^2,\ y(0) = 1.$, Compute y at x = 0.2 by taking h = 0.1 using Runge - Kutta method of order four. (7)

C) Given that $f(z) = u + i\,v$ is analytic and $v(x, y) = -\sin x \sin hy$. Show that $v(x, y)$ is harmonic. Find the conjugate harmonic of $v(x, y)$. (6)

5) A) Find the Laplace transform of $f(t) = e^{-3t}(2\cos 5t - 3\sin 5t)$. (7)

B) Given that $f(z) = u + iv$ is analytic and $u(x, y) = x^3y - xy^3$. (7)

Find the orthogonal trajectories of the family of curves, $u = c$, a constant,

C) Solve $\frac{d^2y}{dx^2} + a^2y = sec\,ax$ by using method of variation of parameters. (6)

6) A) Find the Laplace transform of $\int_0^t \frac{e^t \sin t}{t} dt$. (7)

B) Solve: $(2x + 3)^2\frac{d^2y}{dx^2} - (2x + 3)\frac{dy}{dx} - 12y = 6x.$ (7)

C) Evaluate $\oint_C \frac{dz}{z^2+9}$, where C is $|z + 3i| = 2$, using Cauchy's integral formula. (6)

7) A) Solve: $\frac{dx}{dt} + 2y = -\sin t;\quad \frac{dy}{dt} - 2x = \cos t.$ (7)

B) Find the Laplace transform for the periodic function of period 4, defined by (7)

$$f(t) = \begin{cases} 3t, & 0 < t \le 2 \\ 6, & 2 < t < 4. \end{cases}$$

C) Obtain the Taylor's series expansion of $f(z) = \frac{e^{2z}}{(z-1)^3}$ about z = 1. (6)

8) A) Solve $u_{xx} + u_{xy} - 2u_{yy} = 0$ using the transformation $v = x + y,\ z = 2x - y.$ (7)

B) Rewrite $f(t) = \begin{cases} \cos t, & 0 < t \le \pi \\ 1, & \pi < t \le 2\pi \\ \sin t, & t > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. (7)

C) Using Cauchy's Residue theorem, evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $|z| = 3$. (6)

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