Reg. No.

MANIPAL INSTITUTE OF TECHNOLOGY MANIPAL

IV SEMESTER B.TECH ICE/ EEE ENGINEERING END SEMESTER EXAMINATION, APRIL 2018

SUBJECT: ENGINEERING MATHEMATICS-IV (MAT-2208)

(17-04-2018)

Time: 3 Hours

Max. Marks : 50

1A. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ 0 < x < 1, t > 0 under the given conditions $u(x, 0) = 100 \sin(\pi x)$, $\frac{\partial u}{\partial t}(x, 0) = 0$, u(0, t) = u(1, t) = 0. Compute u for four time steps with $h = \frac{1}{4}$.

1B. In a bolt factory, machines A,B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B?

1C. The mean height of 500 students is 151 cm and standard deviation is 15 cm. Assuming that the heights X are normally distributed, find how many students height lie between 120cm and 155 cm ?

(4+3+3)

2A. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y_0 = y_1 = 0$ using Z-transform.

2B. Solve using finite difference method, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -10(x^2 + y^2 + 10)$ for 0 < x < 3, 0 < y < 3 and u = 0 on its boundary. Take h = 1.

2C. A tea set has four cups and saucers. Two of these are of one colour and the other two are of different colours. If the cups are placed at random on saucers then what is the probability that no cup is placed on a saucer of same colour?

(4+3+3)

3A. Obtain the moment generating function (m.g.f.) of a Poisson distribution with parameter m. Hence find E(X) and V(X).

3B. Find the inverse Z-transform of $F(z) = \frac{z}{(z-1)^2(z+1)^2}$.

3C. Out of 800 families with 5 children each, how many families would be expected to have (a) 3 boys and 2 girls (b) atmost 2 girls by assuming probabilities of births of boys and girls to be equal.

$$(4 + 3 + 3)$$

4A. Solve the BVP y'' + (1+x)y' - y = 0, y(0) = y'(0), y(1) + y'(1) = 0 using finite difference method with the given step size $h = \frac{1}{2}$.

4B. Solve the difference equation $y_{n+2} - 7y_{n+1} + 10y_n = 12e^{3n} + 4^n$.

4C. Find the Z-transform of $f(n) = n(n-1) + \cos\left(\frac{n\pi}{2}\right)$.

(4+3+3)

5A. The random variable (X, Y) has the joint probability distribution given by $f(x, y) = \begin{cases} x + y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{else where.} \end{cases}$ Find the correlation coefficient ρ between X and Y.

5B. Let \bar{X} be the mean of a random sample of size *n* from the population which follows $N(\mu, 9)$. Find *n* such that $P(\bar{X} - 1 < \mu < \bar{X} + 1) = 0.9$.

5C. Let X be a continuous random variable with probability density function $f(x) = \frac{1}{\pi (1+x^2)}$ for $-\infty < x < \infty$. Then find the p.d.f. of $Y = \frac{1}{X}$.

(4+3+3)

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