Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

IV SEMESTER B.TECH (MECHANICAL) END SEMESTER EXAMINATIONS, APRIL-2018

SUBJECT: ENGINEERING MATHEMATICS - IV
[MAT -2210]
REVISED CREDIT SYSTEM

Time: 3 Hours **(19 /04/2018)** MAX. MARKS: 50

Instructions to Candidates:

- ❖ Answer **ALL** questions.

1A. Obtain the series solution of the Bessel’s equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ in the form $y = A\, J_n(x) + B\, J_{-n}(x)$ **4**

1B. Express the polynomial $f(x) = x^3 - 5x^2 + 6x + 1$ in terms of Legendre polynomials. **3**

1C. The random variable X has the following probability function:

x	-2	-1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	K

i) Find the value of K ii) Find the cdf of X iii) Find $P(0 < X < 3)$ **3**

2A. Fit a parabola in the form $y = Ax^2 + Bx + C$ for the following data :

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

4

2B. The chances of A, B, C becoming the general manager of a certain company are in the ratio 4:2:3. The probabilities that the bonus scheme will be introduced in the company if A, B, C become general manager are 0.3, 0.7, 0.8 respectively. If the bonus scheme has been introduced, what is the probability that A has been appointed as general manager? **3**

2C.	Three winning tickets are drawn from an urn containing 100 tickets. What is the probability of winning for a person who buys i) 4 tickets ii) only one ticket?	**3**
3A.	Suppose that the two dimensional random variable (X, Y) has joint pdf $(x) = \begin{cases} e^{-x-y}, & x \geq 0,\ y \geq 0 \\ 0 & elsewhere \end{cases}$. Find the marginal pdfs of X and Y. Are X and Y independent?	**4**
3B.	Show that for the random variable X having normal distribution with mean μ and variance σ^2, $E(X-\mu)^{2n} = 1.3.5.....(2n-1)\sigma^{2n}$.	**3**
3C.	Find the mean and variance of the Poisson distribution with parameter α.	**3**
4A.	Compute an approximate probability that the mean of a random sample of size 25, from a distribution having the pdf $f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & elsewhere \end{cases}$ is between $\frac{1}{4}$ and $\frac{3}{4}$.	**4**
4B.	Two balls are randomly chosen from an urn containing 8 white, 4 black and 2 orange balls. Suppose that we win \$2 for each black ball selected and we loose \$1 for each white ball selected. Let X denote our winnings. Write the probability distribution of X. Find the mean and variance of X.	**3**
4C.	Prove that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$.	**3**
5A.	With the usual notation, prove that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} \mathrm{J_n(x)}t^n$.	**4**
5B.	Suppose that X is a random variable with pdf given by $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$. Find the pdf of $Y = e^{-X}$ and E(Y).	**3**
5C.	The local authorities in a certain city install 10,000 electric lamps in the streets of the city. If these lamps have an average life of 1,000 burning hours with a standard deviation of 200 hours, assuming normality, what number of lamps might be expected to fail i) in the first 800 burning hours? ii) between 800 and 1200 burning hours?	**3**