MANIPAL ACADEMY of HIGHER EDUCATION
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Reg. No.									

DEPARTMENT OF SCIENCES, II SEMESTER M.Sc
(Applied Mathematics and Computing))
END SEMESTER MAKEUP EXAMINATIONS, JUNE2018

Subject [Linear Algebra-MAT 4206]

(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date: .06.2018 MAX. MARKS: 50

Note: (i) Answer all **FIVE FULL** questions

(ii) All questions carry equal marks (4+3+3)

1. (a) Consider the three linear transformations on R^4 given by,

$f(x_1, x_2, x_3, x_4) = x_1 + 2x_2 + 2x_3 + x_4$, $f(x_1, x_2, x_3, x_4) = 2x_2 + x_4$

$f(x_1, x_2, x_3, x_4) = -2x_1 - 4x_3 + 3x_4$. Find the subspace W annihilated by the above transformation.

(b) With usual notation define $L(V, W)$ where V is a n dimensional space and W is a m-dimensional space and exhibit a basis for $L(V, W)$.

(c) If A is a $m \times n$ matrix with entries in a field F, then show that $rowrank(A) = column\ rank(A)$.

2. (a) Let $W_1$and W_2be subspaces of a finite-dimensional vector space V.

(i) Prove that $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$.

(ii) Prove that $(W_1 \cap W_2)^0 = W_1^0 + W_2^0$.

(b) With usual notation show that $\det(AB) = detA.detB$.

(c) Show that, with usual notation $A.(adj\ A) = (Adj\ A).A = \det A.I$.

3. (a) Show that if A is a $m \times n$ matrix and $m < n$, then the homogeneous system of linear equations $AX = 0$ has a non trivial solution. Hence show that, if A is a $n \times n$ matrix, then A is row equivalent to the $n \times n$ identity matrix if and only if the system of equations $AX = 0$ has only trivial solution.

(b) If V is an inner product space, then for any vectors α, β in V, show that

(i) $|< \alpha, \beta >| \leq \|\alpha\| \, \|\beta\|$.

(ii) $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$.

(P.T.O)

(c) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Suppose that V is finite-dimensional, then show that $rank\ (T) + nullity(T) = \dim V$.

4. (a) Let V be a vector space of all $n \times n$ matrices over the field F, and let B be a fixed $n \times n$ matrix. If $T(A) = AB - BA$ verify that T is a linear transformation from V into V.

(b) If f is a non-zero linear functional on the vector space V, then show that the null space of f is a hyperspace in V. Conversely, show that every hyperspace in V is the null space of a (not unique) non-zero linear functional on V.

(c) For a square matrix A, show that sum of its eigenvalues is trace of A and product of the eigenvalues is determinnat of A.

5. (a) Let V be a finite-dimensional vector space over the field F and let $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$be an ordered basis for V. Let W be a vector space over the same field and let $\beta_1, \beta_2, \ldots, \beta_n$be any vectors in W. Then, show that there is precisely one linear transformation T from Vinto W such that $T\alpha_j = \beta_j,\ \ j = 1,2, \ldots, n.$

(b) Let V be a finite-dimensional vector space over the field F and let Tbe a linear operator on V. Then prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1) \ldots (x - c_k)$ where $c_1, \ldots, c_k$ are distinct elements of F.

(c) Let D be an n-linear function on $n \times n$ matrices over K. Suppose D has the property that $D(A) = 0$ whenever two adjacent rows of A are equal. Then D is alternating.

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