MANIPAL ACADEMY of HIGHER EDUCATION
(Deemed to be University under Section 3 of the UGC Act, 1956)

Reg. No.									

DEPARTMENT OF SCIENCES, II SEMESTER M.Sc
(Applied Mathematics and Computing))
END SEMESTER EXAMINATIONS, April 2018

Subject [Linear Algebra-MAT 4206]

(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date: 20.04.2018 MAX. MARKS: 50

Note: (i) Answer all **FIVE FULL** questions
(ii) All questions carry equal marks (4+3+3)

1. (a) Let V be a finite-dimensional vector space over the field F and let $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$be an ordered basis for V. Let W be a vector space over the same field and let $\beta_1, \beta_2, \dots, \beta_n$be any vectors in W. Then, show that there is precisely one linear transformation T from Vinto W such that $T\alpha_j = \beta_j,\ \ j = 1,2, \dots, n$.
   (b) Let V be a vector space which is spanned by a finite set of vectors $\beta_1, \beta_2, \dots, \beta_m$. Then, show that any two independent set of vectors in V is finite and contains no more than m elements.
   (c) Let e be an elementary row operation and E be the $m \times m$ elementary matrix $E = e(I)$. Then, show that for every $m \times n$ matrix A, $e(A) = EA$

2. (a) Let $W_1$and W_2be subspaces of a finite-dimensional vector space V.
      (i) Prove that $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$.
      (ii) Prove that $(W_1 \cap W_2)^0 = W_1^0 + W_2^0$.
   (b) With usual notation of determinant function, show that determinant of a triangular matrix is product of diagonal entries.
   (c) Show that, with usual notation $A.(adj\ A) = (Adj\ A).A = \det A.I$.

3. (a) If A is an $n \times n$ matrix, the prove that, the following are equivalent.
      (i) A is invertible
      (ii) A is row-equivalent to the $n \times n$ identity matrix.
      (iii) A is a product of elementary matrices.
   (b) If V is an inner product space, then for any vectors α, β in V, show that
      (i) $|(\alpha|\ \beta)| \leq \|\alpha\|\ \|\beta\|$.
      (ii) $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$.

(P.T.O)

(c) If W_1 and W_2 are finite-dimensional subspaces of a vector space V, then show that $W_1 + W_2$ is finite dimensional and also show that $\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$.

4. (a) Is there a linear transformation T from R^3 into R^2such that

$T(1,-1,1) = (1,0)$ and $T(1,1,1) = (0,1)$?

If $\alpha_1 = (1,-1), \beta_1 = (1,0)$, $\alpha_2 = (2,-1), \beta_2 = (0,1), \alpha_3 = (-3,2)$,

$\beta_3 = (1,1)$, is there a linear transformation T from R^2 into R^2 such that $T\alpha_i = \beta_i$ for $i = 1,2$ and 3?

(b) If f is a non-zero linear functional on the vector space V, then show that the null space of f is a hyperspace in V. Conversely, show that every hyperspace in V is the null space of a (not unique) non-zero linear functional on V.

(c) For a square matrix A, show that sum of its eigenvalues is trace of A and product of the eigenvalues is determinnat of A.

5. (a) Let R be the field of real numbers, and let D be a function on 2×2 matrices over R, with values in R, such that $D(AB) = D(A)D(B)$ for all A, B. Suppose also that $D\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) \neq D\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$. Prove the following.

(i) $D(0) = 0$; (ii) $D(A) = 0$ if $A^2 = 0$;
(iii) $D(B) = -D(A)$ if B is obtained by interchanging the rows (or columns) of A.

(b) Let V be a finite-dimensional vector space over the field F and let Tbe a linear operator on V. Then prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1) \dots (x - c_k)$ where $c_1 , \dots , c_k$ are distinct elements of F.

(c) Let $n > 1$ and let D be an alternating $(n-1)$-linear function on $(n-1) \times (n-1)$ matrices over K. Prove that for each $j, 1 \leq j \leq n$, the function $E_j$defined by $E_j(A) = \sum_{i=1}^{n} (-1)^{i+j} A_{ij} D_{ij}(A)$ is an alternating n-linear function on $n \times n$ matrices. Also prove that if D is a determinant function, so is each E_j.

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