Reg. No.										

DEPARTMENT OF SCIENCES, IV SEMESTER M.Sc (Applied Mathematics & Computing)
END SEMESTER EXAMINATIONS, APRIL 2018

STOCHASTIC PROCESSES AND RELIABILITY [MAT 706]

(REVISED CREDIT SYSTEM)

Time: 3 Hours   Date:23/04/2108   MAX. MARKS: 50

Note: (i) Answer any **FIVE FULL** questions. All questions carry equal Marks **(3 + 3+ 4)**
(ii) Draw diagrams, and write equations wherever necessary

1A. Define generating function, when does generating function becomes p.g.f. ?

Consider the process $X(t) = A \cos wt + B \sin wt$, where A ,B are uncorrelated random variables with mean 0 and variance 1 and w is a positive constant. Is X(t) covariance stationary?

1B. If N(t) is a Poisson process and s<t, then find the probability of $P\{N(s)=k|N(t)=n\}$.

1C. Suppose that E and F occur independently and in accordance with Poisson processes with parameters a and b respectively. Find the probability that k occurrences of E take place between every second occurrence of F .

2A. Three children (denoted by 1,2,3) arranged in a circle play a game of throwing a ball to one another. At each stage the child having the ball is equally likely to throw it to any one of the other two children. Suppose that X_0 denotes the child who had the ball initially and $X_n(n \geq 1)$ denotes the child who had the ball after n throws.Show that$\{X_n\}$ forms a Markov chain. Evaluate (i)T.p.m P (ii) $P\{X_2=2, X_1=1 \mid X_0=2\}$.(iii) $P\{X_2=2, X_1=1, X_0=2\}$.

2B. State Chapman – Kolmogorov's equation.

2C. A particle starting from the origin moves from position j to position (j+1) with probability a_j and returns to origin with probability (1- a_j).Suppose that the states , after n moves are 0,1,2,……then show that the state 0 is recurrent iff $\lim(a_1,\ldots,a_n) \to 0$ as $n \to \infty$.

3A. A person enlists subscriptions to a magazine, the number enlisted being a Poisson process with mean rate 6 per day. Subscribers may subscribe for 1 or 2 years independently of one another with respective probabilities 2/3 and 1/3.Find the total commission earned in period t and its variance.

3B. Consider a Yule-Furry process starting with a single member at time t = 0, and having birth rate λ. Suppose that this first member(parent) is also subject to death, his lifetime being distributed as an exponential variable with parameter μ. Find the distribution of the number N of offspring due to this parent as well as his descendants at the time of death of the parent.

3C. The number of accidents in a town follows a Poisson process with a mean of two per day and the number X_i of people involved in the i^{th} accident has the distribution $P\{X_i=k\}=1/2^k (k\geq 1)$.Find the mean and variance of the number of people involved in accidents per week.

4A. Find the differential equation of pure death process. If the process starts with i individuals, find the mean and variance of the number N(t) present at time t.

4B. Find the probability of ultimate extinction in the case of the linear growth process starting with 100 individuals at time 0.

4C. Find the generating function of the sequence of Fibonacci numbers $\mathbf{f_0 = 0, f_1 = 1, f_n = f_{n-1} + f_{n-2}}$ for $n \geq 2$.

5A. Consider the Markov chain with t.p.m

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 0 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 2 & 0 & 1 & 0 \end{array}$$

Find the mean recurrence time for the state 2. Is the chain irreducible, ergodic? If so find the limiting distributions.

5B. Suppose that customers arrive at a counter with a mean rate of 2/min. Find the probability that the interval between two successive arrival is

(i) more than 2 min (ii) 5 min or less (iii) between 2 & 6 min.

5C. State and derive Yule – Fury birth process.

6A. Prove that the state j is persistent or transient according as $\sum_{n=0}^{\infty} P_{jj}^{(n)} = \infty$ or $< \infty$.

6B. Suppose that a fair die is tossed. Let the states of X_n be k (=1,2,.....,6),where k is the maximum number shown in the first n tosses. Find p and $\boldsymbol{\mu}_{jj}$,for j= 1,4.Is the state 3 absorbing?

6C. Define the followings:

(i) Immigration – Emigration Process (ii) Immigration – Death Process

(iii) Time dependent Poisson Process

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