MANIPAL ACADEMY of HIGHER EDUCATION
(Deemed to be University under Section 3 of the UGC Act, 1956)

Reg. No.										

DEPARTMENT OF SCIENCES, IV SEMESTER M.Sc (Applied Mathematics & Computing)
END SEMESTER MAKEUP EXAMINATIONS, June 2018

STOCHASTIC PROCESSES AND RELIABILITY [MAT 706]

(REVISED CREDIT SYSTEM)

Time: 3 Hours   Date:11/06/2018   MAX. MARKS: 50

Note: (i) Answer any **FIVE FULL** questions. All questions carry equal Marks **(3 + 3+ 4)**
(ii) Draw diagrams, and write equations wherever necessary

1A.	Let $X(t)=\sum_{r=1}^{k}\left(A_r\cos\theta_r t+B_r\sin\theta_r t\right)$ where A_r, B_r are uncorrelated random variables with mean 0, variance σ^2 and θ_r are constants . Is {X(t)} covariance stationary? Is it a Markov Process?
1B.	Solve the difference equation using p.g.f.: $u_n = qu_{n-1} + p(1-u_{n-1}), n \geq 1, p+q=1, u_0 = 1.$
1C.	Suppose that the probability of a rainy day (state 1) following a dry day(state 0) is 1/3 and that the probability of a rainy day following a dry day is ½.Given that May 1 is a dry day, what is the probability May 3 is a dry day?
2A.	Show that a Markov chain is completely specified by the transition matrix and the initial distribution.
2B.	Divide the interval [0,t] into a large number n of small intervals of length h and suppose that in each small interval, Bernoulli trials with probability of successes λh are held. Show that the number of successes in an interval of length t is a Poisson process with mean λt. State the assumptions you make.
2C	Define an intree to a specific point. Explain graph theoretic approach to evaluate limiting distribution to the t.p.m $P=\begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$

3A	Let X have a zero – truncated Poisson distribution with zero class missing. Find the p.g.f of X and $E(X^2)$..
3B	Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0,1,2 and with transition Matrix. $\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$ and the initial distribution $Pr\{X_0 = i\} = 1/3, i = 0,1,2$ Evaluate : (i) $Pr\{X_2 = 2, X_1 = 1 \| X_0 = 2\}$ (ii) $Pr\{X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2\}$
3C	Prove that in an irreducible chain , all the states are of the same type.
4A	Consider a sequence $\{X_n\}$ of independent coin-tossing trials with probability p for head H in a trial. Denote the states of X_n by states 1, 2, 3, 4 according as the trial numbers (n-1) and result in HH,HT,TH,TT respectively. Show that $\{X_n\}$ is a Markov chain Find (i) P ($X_1 = 3 / X_3 = 3$) (ii) P ($X_3 = 3 / X_1 = 4$).
4B	Find the probability of ultimate extinction in the case of the linear growth process starting with *i* individuals at time 0.
4C	Prove that sum of two independent Poisson processes is a Poisson process.
5A	A radioactive source emits particles at a rate of 5 per minute in accordance with a Poisson Process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that the number of particle recorded is 10 in a interval of 4 minute.
5B	If N(t) is a poisson process then prove that auto correlation coefficient between N(t) and N(t+s) is $\{t/t+s\}^{1/2}$.
5C	Suppose that a certain system can be considered to be in two states : "Operating" and "Under repair" denoted by 1 and 0 respectively. Suppose that the lengths of operating period and the period under repair are independent random variables having negative exponential distributions with means 1/2 and 1/3 respectively. Find the probabilities for two states for 0 and 1.(i.e.$p_{ij}(t)$, for i,j=0,1).
6A	State and prove chapman – Kolomogorov's equation.
6B	Under the postulates of Poisson process, prove that N(t) follows Poisson distribution with mean λt.
6C	Suppose that customers arrive at a counter with a mean rate of 2/min. Find the probability that the interval between two successive arrival is (i) more than 2 min (ii) 5 min or less (iii) between 2 & 6 min.