



## DEPARTMENT OF SCIENCES, M. Sc. (Physics) II SEMESTER, END SEMESTER EXAMINATIONS APRIL 2018 Subject: Quantum Mechanics II (PHY-4206) (REVISED CREDIT SYSTEM - 2017)

Time: 3 Hours Date: April 2018 MAX. MARKS: 50

Note: (i) Answer all the questions. (ii) Answer the questions to the point.

1. (i) A particle is in an eigenstate of  $L_z$ . Prove that  $\langle J_x \rangle = \langle J_y \rangle = 0$ . Also find the value of  $\langle J_x^2 \rangle$  and  $\langle J_y^2 \rangle$ . [5]

(ii) Using the WKB method, calculate the transmission coefficient for the potential barrier

$$V(x) = \begin{cases} V_0 \left( 1 - \frac{|x|}{\lambda} \right), & |x| < \lambda \\ 0, & |x| > \lambda. \end{cases}$$
[5]

2. (i) The time varying Hamiltonian H'(t) induces transitions between states  $|j\rangle$  and  $|k\rangle$ . Using time-dependent perturbation theory, show that the probability for a transition from state  $|j\rangle$  to state  $|k\rangle$  is the same as the probability for a transition from state  $|k\rangle$  to state  $|j\rangle$ . [5]

(ii) A quantum mechanical system is initially in the ground state  $|0\rangle$ . At t = 0, a perturbation of the form H'(t),  $H_0 e^{-\alpha t}$ , where  $\alpha$  is a constant, is applied. Calculate the probability of finding the system in state  $|1\rangle$  after long time. [5]

3. (i) Estimate the energy levels of a particle moving in the potential

$$V(x) = \begin{cases} \infty, & x < 0; \\ Ax, & x > 0. \end{cases}$$

5

where A being a constant.

(ii) Calculate the scattering amplitude for a particle moving in the potential

$$V(r) = V_0 \frac{c-r}{r} exp\left(-\frac{r}{r_0}\right)$$

where  $V_0$  and  $r_0$  are constants.

4. Obtain the expression of integral form of the Schoedinger equation. [10]

[5]

5. (i) Show that the Dirac matrices are traceless and can be of even order only. [5]

(ii) Show that the matrix  $\vec{\sigma'} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$  is not a constant of motion. [5]

Useful formulae:

$$\nabla^{2}t = \frac{1}{r^{2}}\frac{\partial}{\partial r}\left(r^{2}\frac{\partial t}{\partial r}\right) + \frac{1}{r^{2}sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^{2}sin^{2}\theta}\frac{\partial^{2}t}{\partial \phi^{2}}$$
$$\int_{0}^{\infty}exp(-a^{2}x^{2})cos(bx)\,dx = \frac{\sqrt{\pi}}{2a}exp\left(-\frac{b^{2}}{4a^{2}}\right)$$
$$\int_{0}^{\infty}x^{n}exp(-ax)\,dx = \frac{n!}{a^{n+1}}, \quad \text{where} \quad n \ge 0, \quad a > 0$$