

Table: Q.5A

$x_1$	$x_2$	$x_3$	$R_1$	$R_2$
2.3	1.0	0.5	0.0	1.0

5A. For the data shown in Table Q.5A, show the first iteration of back propagation algorithm in trying to compute the membership values for the input variables  $x_1$ ,  $x_2$  and  $x_3$  in the output region  $R_1$  and  $R_2$ . Use a  $3 \times 3 \times 1$  neural network. Assume a random set of weights for your neural network. [5]

5B. When gyros are calibrated for axis bias, they are matched with a temperature. Thus, we can have a relation of gyro bias (GB) vs. temperature (T). Suppose we have fuzzy sets for a given gyro bias and a given Fahrenheit temperature as follows:

$$\mu_{GB}(x) = \left\{ \frac{0.2}{3} + \frac{0.4}{4} + \frac{1}{5} + \frac{0.4}{6} + \frac{0.2}{7} \right\} \quad \text{bias in degrees Fahrenheit per hour}$$

$$\mu_T(y) = \left\{ \frac{0.4}{66} + \frac{0.6}{68} + \frac{1}{70} + \frac{0.6}{72} + \frac{0.4}{74} \right\} \quad \text{temperature in degrees Fahrenheit}$$

- Use a Mamdani implication to find the relation IF gyro bias, THEN temperature.
- Suppose we are given a new gyro bias (GB') as follows:

$$\mu_{GB'}(x) = \left\{ \frac{0.6}{3} + \frac{1}{4} + \frac{0.6}{5} \right\}$$

Use max-min composition, find the temperature associated with this new bias. [3]

5C. This problem deals with the voltages generated internally in switching power supplies. Embedded systems are often supplied 120 V AC for power. A "power supply" is required to convert this to a useful voltage (+5 V DC). Some power supply designs employ a technique called "switching." This technique generates the appropriate voltages by storing and releasing the energy between inductors and capacitors. This problem characterizes two linguistic variables, high and low voltage, on the voltage range of 0 to 200 V AC:

$$\text{"High"} = \left\{ \frac{0}{0} + \frac{0}{25} + \frac{0}{50} + \frac{0.1}{75} + \frac{0.2}{100} + \frac{0.4}{125} + \frac{0.6}{150} + \frac{0.8}{175} + \frac{1}{200} \right\}$$

$$\text{"Medium"} = \left\{ \frac{0.2}{0} + \frac{0.4}{25} + \frac{0.6}{50} + \frac{0.8}{75} + \frac{1}{100} + \frac{0.8}{125} + \frac{0.6}{150} + \frac{0.4}{175} + \frac{0.2}{200} \right\}$$

Find the membership function for the phrase, "very, very high or very, very medium." [2]



**Instructions to candidates**

- Answer ALL FIVE full questions. All questions carry equal marks.
- Missing data if any, may be suitably assumed.

1A. Describe the following learning techniques:

- Error-correction learning
- Memory-based learning
- Competitive-learning, and
- Boltzmann learning.

[5]

1B. Consider the following orthonormal sets of key patterns, applied to a correlation matrix memory:

$$x_1 = [1, 0, 0, 0]^T \quad x_2 = [0, 1, 0, 0]^T \quad x_3 = [0, 0, 1, 0]^T$$

The respective stored patterns are:

$$y_1 = [5, 1, 0]^T \quad y_2 = [-2, 1, 6]^T \quad y_3 = [-2, 4, 3]^T$$

- Calculate the memory matrix  $M$
- Show that the memory associates perfectly.

[3]

1C. A Bayes classifier for Gaussian distributed data has following relation

$$\log \Lambda(x) = (\mu_1 - \mu_2)^T C^{-1} x + \frac{1}{2} (\mu_2^T C^{-1} \mu_2 - \mu_1^T C^{-1} \mu_1)$$

, where  $\log \Lambda(x)$  is the log-likelihood ratio of conditional density function. Now, consider two one-dimensional, Gaussian-distributed classes  $C_1$  and  $C_2$  that have a common variance equal to 1. Their mean values are  $\mu_1 = -10$ , and  $\mu_2 = +10$  respectively. These two classes are essentially linearly separable. Design a classifier that separates these two classes. [2]

2A. Design a polynomial learning machine, whose inner-product kernel is given by

$$K(x, x_i) = (1 + x^T x_i)^2$$

to solve the XOR problem. Also compute the optimum margin of separation  $\rho$ . [5]

2B. Explain various heuristics for making the back-propagation algorithm perform better.

[3]

2C. Express interpolation problem in strict sense in terms of *radial-basis function*.

[2]

3A. Given a set of measurements of the magnetic field near the surface of a person's head, we want to locate the electrical activity in the person's brain that would give rise to the measured magnetic field. This is called the inverse problem, and it has no unique solution. One approach is to model the electrical activity as dipoles and attempt to find one to four dipoles that would produce a magnetic field closely resembling the measured field. For this problem we will model the procedure a neuroscientist would use in attempting to fit a measured magnetic field using either one or two dipoles. The scientist uses a reduced chi-square statistics to determine how good the fit is. If  $R = 1.0$  the fit is exact. If  $R \geq 3$ , the fit is bad. The range of  $R$  will be taken as  $R = \{1.0, 1.5, 2.0, 2.5, 3.0\}$  and we define the following fuzzy sets for  $D_1$  = one-dipole model, and  $D_2$  = two-dipole model:

$$D_1 = \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\} \quad D_2 = \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

For these two fuzzy sets, find the following:

- (i)  $D_1 \cup D_2$                       (iii)  $\overline{D_1 \cup D_2}$                       (v)  $D_2 \cap \overline{D_2}$   
(ii)  $D_1 | D_2$                       (iv)  $\overline{D_1 \cap D_2}$                       (vi)  $\overline{D_1} \cup D_1$ .

[5]

3B. In photography, it is important to relate reagent thickness to color balance on the film medium. Let  $Y$  be a universe of color balance,  $Y = [0, 1, 2, 3, 4]$ , where 0=yellow, 4=blue, and 2=neutral. Let  $X$  be a universe of the reagent thickness,  $X = [0, 1, 2, 3, 4]$ , where 0=thin, 4=thick, and 2=semi-thick. Now, suppose that a relation is obtained from a Cartesian product as follows:

$$R = X \times Y = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0.8 & 0.6 & 0.2 & 0 \\ 0.8 & 1 & 0.8 & 0.6 & 0 \\ 0.6 & 0.8 & 1 & 0.8 & 0.6 \\ 0.2 & 0.6 & 0.8 & 1 & 0.8 \\ 0 & 0.2 & 0.6 & 0.8 & 1 \end{bmatrix} \end{matrix}$$

It is required to relate color balance on the film medium to the perceived quality of the picture. For this we need an additional universe of perceived picture quality,  $Z = [0, 1, 2, 3, 4]$ ; where 0=bad, 4=excellent, and 2=fair. Suppose a relation is obtained from a Cartesian product

$$S = Y \times Z = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0.6 & 0.4 & 0.2 & 0 \\ 0.6 & 1 & 0.6 & 0.4 & 0 \\ 0.4 & 0.6 & 1 & 0.6 & 0.4 \\ 0.2 & 0.4 & 0.6 & 1 & 0.6 \\ 0 & 0.2 & 0.4 & 0.6 & 1 \end{bmatrix} \end{matrix}$$

i) Find  $T = R \circ S$  using max-min composition

ii) Find  $T = R \circ S$  using max-product composition.

[3]

3C. Consider a fuzzy set  $A$ , mathematically define the following properties of the membership function  $\mu_A(x)$ :

- i) Core of the membership function  
ii) Support of the membership function  
iii) Boundaries of the membership function  
iv) Normal fuzzy set.

[2]

4A. Many products, such as tar, petroleum jelly, and petroleum, are extracted from crude oil.

In a newly drilled oil well, three sets of oil samples are taken and tested for their viscosity. The results are given in the form of the three fuzzy sets  $A_1$ ,  $A_2$ , and  $A_3$ , all defined on a universe of normalized viscosity, as shown in Figure Q.4A. Use centroid method to find the most nearly representative viscosity values of all three oil samples.

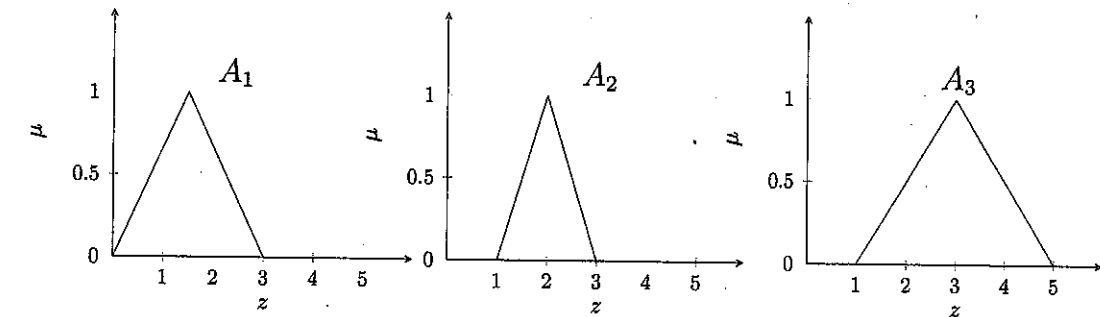


Figure: Q.4A

[5]

4B. Two fuzzy sets  $A$  and  $B$ , both defined on  $X$ , are given in Table Q.4B. Show that the

Table: Q.4B

$\mu(x_i)$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$
$\tilde{A}$	0.1	0.6	0.8	0.9	0.7	0.1
$\tilde{B}$	0.9	0.7	0.5	0.2	0.1	0

$\lambda$ -cut set obey the following properties:

- i)  $(\tilde{A} \cup \tilde{B})_{0.3} = A_{0.3} \cup B_{0.3}$   
ii)  $(\tilde{A} \cap \tilde{B})_{0.4} = A_{0.4} \cap B_{0.4}$   
iii)  $(\tilde{A})_{0.6} \neq \overline{A}_{0.6}$ .

[3]

4C. Using the inference approach, find the membership values for each of the triangular shapes (I,R,IR,E,T) for the triangle,  $120^\circ, 50^\circ, 10^\circ$ .

[2]