# Question Paper

Exam Date & Time: 13-Nov-2018 (02:00 PM - 05:00 PM)

## MANIPAL ACADEMY OF HIGHER EDUCATION

### INTERNATIONAL CENTRE FOR APPLIED SCIENCES
### I SEMISTER B.Sc. APPLIED SCIENCES THEORY EXAMINATION- NOV 2018
### MATHEMATICS - 1 [IMA 111]

**Marks: 100** **Duration: 180 mins.**

## Answer 5 out of 8 questions.

1) (8)

Find the $n^{th}$ derivative of the following

A)

i). $\dfrac{x+3}{(x-1)(x+2)}$, ii). $\sin^3 x$

B) (6)

Obtain reduction formula for $\int \sin^n x\,dx$

and hence evaluate $\int_0^{\pi/2} \sin^n x\,dx$

C) (6)

State Leibnitz's theorem. If $y = \sin^{-1} x$, show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$$

2) (8)

Evaluate the following

A)

i). $\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$, ii). $\lim_{x \to 0} \dfrac{e^x - e^{-x} - x}{x^2 \sin x}$

B) (6)

Find the volume of the solid generated by revolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis

C) Find in symmetrical form, the equations of (6)

the line $x + y + z + 1 = 0 = 4x + y - 2z + 2$

3) Evaluate the following (8)

A)

i). $\int_0^1 x^{3/2}(1-x)^{3/2}\,dx$, ii). $\int x^6 \sin^{-1} x\,dx$

B) State Cauchy's mean value theorem. Verify the (6)

Cauchy's mean value theorem for the functions $x^2$ and $x^4$ in the positive interval [a, b]

C) Find the equation of the plane which bisects the line (6)

segment joining the points (-8, 3, 7) and (2, 3, 1) at right angles

4) Trace the curve $y^2(a - x) = x^3$, $a > 0$ with explanations (8)

A)

B) (6)

Show that $\frac{3\sqrt{3}}{2}$ is the least value of $|\rho|$ for $y = \log x$

C) Find the radius of curvature of $x = a(t + \sin t)$ (6)

$y = a(1 - \cos t)$ at any t.

5) (8)

A)

Evaluate the following

i). $\lim_{x \to 0}(\text{Cot}^2 x - \frac{1}{x^2})$ ii). $\lim_{x \to 0}(1+x)^{1/x}$

B) Obtain the Evolute of the parabola $y^2 = 4ax$ (6)

C) Define directional ratios of a line. Find the angle between the diagonals of a cube (6)

6) State D'Alembert's ratio test. Discuss the convergence of the series, (8)

A)

i). $\frac{x}{1(2)} + \frac{x^2}{2(3)} + \frac{x^3}{3(4)} + \ldots + \frac{x^n}{n(n+1)} + \ldots\infty$

ii). $\sum_{1}^{\infty} \frac{n!}{n^n}$

B) Find an equation to the plane through the points (3, 2, 1), (3, 2,4) and perpendicular to the plane $2x + 3y - z + 5 = 0$ (6)

C) Find the equation of the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $(\frac{a}{4}, \frac{a}{4})$. (6)

7) (8)

A)

Define conditionally and absolutely convergece series. Check the series for absolutely convergence

i). $1-\frac{1}{2}+\frac{1}{2^2}\ldots\ldots\ldots\ldots+\frac{(-1)^n}{2^{n-1}}+\ldots\ldots\ldots\infty$

ii). Define Raabe's test. Test for convergence of the series $\sum_{1}^{\infty}\frac{3^n\,n!}{n^n}$

B) (6)

Find the distance of the point (3,4,5) from the plane $2x+3y+5y-6z-7=0$, measured parallel to the plane $\frac{x}{2}=\frac{y}{2}=\frac{z}{-2}$

C) (6)

State Newton forward interpolation formulae. Find a cubic polynomial for the following data (-1, 0), (0, 1), (1, 2), (2, 9).

8) (8)

A) i). Find a cubic polynomial for the following values (0,1) (1, 0), (2, 1), (4, 33).

ii). Find an equation to a plane parallel to the plane $2x-3y+z-5=0$ and passing through the point (2,-1,4)

B) (6)

Find the equation of the Right circular Cylinder of radius 2 units , whose axis is the Z-axis.

C) Apply Maclaurin's series to find the expansion of $\log(1 + \sin x)$. (6)

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