Question Paper

Exam Date & Time: 13-Nov-2018 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATION NOVEMBER - 2018
THIRD SEMISTER B Sc. (Applied Sciences) in Engg.
Mathematics - I [MA 111]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.

1) (8)

1A) Find the n^{th} derivative of the following

a) $\dfrac{15x-14}{(x-1)(x-2)}$, b) $e^{-x}\cos^2 x$

1B) (6)

Obtain reduction formula for $\int \cos^n x\,dx$ and hence evaluate $\int_0^{\frac{\pi}{2}} \cos^n x\,dx$

1C) (6)

State Leibnitz's theorem. If $y=\tan^{-1}x$, prove that

$$(1+x^2)y_{n+2}+2(n+1)xy_{n+1}+n(n+1)y_n=0$$

2) (8)

2A) Evaluate the following

i). $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{e^x-1}\right)$, ii). $\lim\limits_{x\to 0}(\cot x)^{\sin x}$

2B) (6)

Find the volume of the solid obtained by revolving one arch of the cycloid $x=5(t+\sin t)$, $y=5(1-\cos t)$ about x-axis

2C) (6)

Find the equation of the right circular cone whose vertex is at the origin and semi-vertical angle is α and having axis of Z as its axis

3) (8)

3A) Evaluate the following

i). $\int_0^1 x^6 (1-x^2)^{\frac{3}{2}} dx$ ii). $\int_0^{2\pi} \sin^4 x \cos^5 x dx$

3B) Define the Radius of Curvature of a curve at any point P. With the usual notation find the radius of curvature of the curve y = f(x). (6)

3C) State Cauchy's mean value theorem. Verify the Cauchy'smean value theorem for the functions x^2 and x^4 in the positive interval [a, b] (6)

4) (8)

4A) Find the radius of curvature of x =a(t + sint), y = a(1 - cost) at any t.

4B) Show that the Cardioids $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ intersect orthogonally (6)

4C) Trace the curve $y^2(a - x) = x^3$, $a \succ 0$ with explanations (6)

5) (8)

5A) Obtain the Evolute of the curve

$$x = a(\cos t + \log\tan(\frac{t}{2})),\ y = a\sin t$$

5B) (6)

Define directional ratios of a line. Find the angle between the diagonals of a cube

5C) (6)

Find the centre and radius of the circle

$$x^2 + y^2 + z^2 - 2y - 4z - 11 = 0, x + 2y + 2z - 15 = 0$$

6) (8)

6A) State Cauchy's root test. Discuss the convergence of the series,

1). $\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \ldots\ldots\ldots\ldots + \frac{1}{n(n+1)} + \ldots\ldots\ldots\infty$

ii). $\sum_{1}^{\infty}\left(\frac{2}{3}\right)^n$

6B) (6)

Find the equation of the right circular cone whose vertex is the origin, whose axis is line passing through the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has semivertical angle 30^0

6C) (6)

Find the equation of the circle of curvature of the curve

$\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $(\frac{a}{4}, \frac{a}{4})$.

7) (8)

7A) Define conditionally and absolutely convergent series. Check the series for absolutely convergent,

i). $1 - \frac{1}{2} + \frac{1}{3} \ldots\ldots + \frac{(-1)^{n-1}}{n} + \ldots\ldots\ldots\infty$

ii). Using Lagranges Interpolation formula, find a second degree polynomial for the following data (0, 1), (1, -1), (3, 7)

7B) Find the distance of the point (3,4,5) from the plane (6)

2x+3y+5y-6z-7 = 0, measured parallel to

the plane $\frac{x}{2} = \frac{y}{1} = \frac{z}{-2}$

7C) (6)

Find the volume generated by the revolution of

the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the major axis

8) (8)

8A) State Leibnitz's rule. Find the interval of convergence of the series

i). $1 + x + x^2 + x^3 + \ldots\ldots\ldots\ldots\infty$

ii). Find an equation to a plane parallel to the plane 2x -3y+z -5 =0 and passing through the point (2,-1,4)

8B) (6)

Trace the curve $r = 2(1 + \cos\theta)$,

Find the entire length of the curve

8C) (6)

Apply Maclaurin's series to find the expansion of $\log(1 + \sin x)$.

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