Question Paper

Exam Date & Time: 28-Nov-2018 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLED SCIENCES
II SEMESTER B.Sc. APPLIED SCIENCES IN ENGINEERING END SEMESTER EXAMINATION-NOVEMBER/DECEMBER 2018

Mathematics - II [IMA 121]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.

1) A) Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at $(1, -2, 1)$. (7)

B) Test whether the set $B = \{(1,\ 1, 0),\ (3,\ 0,\ 1),\ (5, 2, 2)\}$ forms a basis for R^3. If so represent $(1,\ 2, 3\ )$ in terms of basis vectors (7)

C) The period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$, find the maximum error in T due to the possible error upto 1% in l and 2.5% in g. (6)

2) A) Evaluate $\iint_R (xy - y^3)\, dy\, dx$ where $R = \{(x, y){:}\, 0 < x < y < 1\}$. (7)

B) Determine whether $\vec{F} = (x + 2y + 4z)i + (2x - 3y - z)j + (4x - y + 2z)k$ is conservative? If so find scalar potential. (7)

C) State Euler's theorem for homogeneous function of degree n.
If $u = cosec^{-1}\left[\frac{\sqrt{x}+\sqrt{y}}{\sqrt[3]{x}+\sqrt[3]{y}}\right]^{1/2}$ prove that $\frac{\partial u}{\partial x} + y\,\frac{\partial u}{\partial y} = -\,\frac{1}{12}\, tanu$. (6)

3) A) Test for consistency and solve $5x + 3y + 7z = 4, 3x + 26y + 2z = 9,$ $7x + 2y + 10z = 5.$ (7)

B) Calculate the volume of the solid bounded by the planes $x = 0,\ \ y = 0,$ $x + y + z = a$ and $z = 0.$ (7)

C) Prove: (a) $\nabla \times (\nabla\phi) = 0$ (b) $\nabla \cdot (\nabla \times A) = 0.$ (6)

4) A) Divide 24 as a sum of 3 numbers such that the continued product of the first, square of the second and cube of the third is maximum. (7)

B) (7)

Evaluate the integral by changing to polar coordinates

$\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\ dy\,dx$

C) Using Gauss - Jordan method, find the inverse of the matrix $\begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$. (6)

5) (7)

A) Find the maximum and minimum values of the function $f(x,y) = x^3 + y^3 - 3axy.$

B) If $u = u(\frac{y-x}{xy}, \frac{z-x}{xz})$, show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$ (7)

C) Determine the values of p such that the rank of $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ p & 2 & 2 & 2 \\ 9 & 9 & p & 3 \end{bmatrix}$ is 3. (6)

6) (7)

A) Evaluate (i) $\int_0^\infty \sqrt{x}\ \ e^{-x^3}dx$ (ii) $\int_0^1 x^7\,(1-x^4)^3\,dx$.

B) Find the total work done in moving a particle in the force field given by $\vec{F} = zi + zj + xk$ along the helix C given by $x = cost,\ y = sint, z = t$ From $t = 0$ to $t = \frac{\pi}{2}$. (7)

C) Using Gram Schmidt process construct an orthonormal set of basis vectors of R^3 for given vectors $\{(1,\ 1,\ 1),\ (-1,\ 0,\ -1),\ (-1,\ 2,\ 3)\}$. (6)

7) (7)

A) Change the order of integration and evaluate $\int_0^1 \int_{\sqrt{y}}^{2-y} xy\,dx\,dy$.

B) Verify Green's theorem for $\oint_C (3x^2 - 8y^2)dx + (4x - 6xy)dy$ where C is the boundary of the region bounded by $x = 0, y = 0$ and $x + y = 1$. (7)

C) If $z = e^{ax+by}f(ax-by)$, prove that $b\,\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz.$ (6)

8) (7)

A) Use Gauss Divergence theorem to evaluate $\iint_S \vec{f}\ .n\,ds$, where $\vec{f} = 4xi - 2y^2j + z^2k$ and S is the surface bounding the region $x^2 + y^2 = 4,\ z = 0$ and $z = 3$.

B) Find the area lying inside the circle $r = asin\theta$ and outside the cardioid $r = a(1 - cos\theta)$. (7)

C) Let $\phi = x^2yz - 4xyz^2$. Find the directional derivative of ϕ at $P(1, 3, 1)$ in the direction of $2i - j - 2k$. (6)

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