Question Paper

Exam Date & Time: 28-Nov-2018 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLED SCIENCES
II SEMESTER B.S. ENGINEERING END SEMESTER EXAMINATION-NOVEMBER/DECEMBER 2018
Mathematics - II [MA 121]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.
Missing data, if any, may be suitably assumed

1)

A) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ by changing the order of integration. (7)

B) If $z(x+y) = (x^2+y^2)$ show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$. (7)

C) Check if $B = \{(3,0,2),\ (7,0,9),\ (4,\ 1,2)\}$ forms a basis of R^3. If so express $(1,2,3)$ in terms of basis vectors. (6)

2)

A) Test for consistency of the following equations and find the solutions of $4x - 2y + 6z = 8,\quad x + y - 3z = -1,\ \ 15x - 3y + 9z = 21.$ (7)

B) Find by the double integration, the area lying inside the cardioid $r = a(1 + cos\theta)$ and outside the circle $r = a$. (7)

C) Find the equation of the tangent plane to the surface $x^2yz - 4xyz^2 = -6$ at $(1,2,1)$. (6)

3)

A) If $u = tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$ show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\, cos3u\, sinu$ (7)

B) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$ by changing to polar co-ordinates. (7)

C) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & 1 & 6 \end{bmatrix}$ (6)

4) Show that $\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$ (7)

A)

B) Find the extreme value of $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ (7)

C) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 1 \\ -1 & 1 & 2 \\ 2 & 1 & -2 \end{bmatrix}$ using Gauss Jordan method. (6)

5) A) Find the maximum and minimum distances of the point $A = (1,-1,2)$ from the sphere $x^2 + y^2 + z^2 = 9$. (7)

B) Apply green's theorem to evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$. (7)

C) Using Gram –Schmidt process construct an orthonormal basis from the set of vectors $\{(3,0,4),\ (-1,\ 0,\ 7),\ (2,\ 9,11)\}$ in R^3. (6)

6) A) Find the work done in moving a particle in the force field $\vec{F} = 3x^2 i\ (2xz - y)j + zk$ along the curve defined by $x^2 = 4y,\ 3x^3 = 8z$ from $x = 0$ to $x = 2$. (7)

B) Evaluate $(i)\ \int_0^\infty x^6 e^{-2x}\, dx$ (ii) $\int_0^1 x^2 \left(\log\frac{1}{x}\right)^3 dx$ (7)

C) Prove that $\nabla \times (\emptyset\vec{A}) = (\nabla\emptyset) \times \vec{A} + \emptyset(\nabla \times \vec{A})$. (6)

7) A) Evaluate $\iint_R r\, dr d\theta$ over the region R is bounded $x^2 + y^2 = 2ax$ and $y = x$ in the first quadrant (7)

B) Let $u = f(r)$, where $r = \sqrt{x^2 + y^2}$ then, prove that $u_{xx} + u_{yy} = f''(r) + \frac{1}{r} f'(r)$. (7)

C) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. (6)

8) A) Evaluate $\iint_S \vec{A} . \vec{n}\, ds$ where $\vec{A} = 18zi - 12j + 3yk$ and S is part of plane $2x + 3y + 6z = 12$ which is located in first octant. (7)

B) Calculate the volume of the solid bounded by the planes $x = 0,\ y = 0,$ $x + y + z = a$ and $z = 0$. (7)

C) (6)

The diameter and altitude of a can in the shape of a right circular cylinder are measured as 4cm and 6cm respectively. The possible error in each measurement is 0.1cm. Find approximately the maximum possible error in the values computed for the volume and the lateral surface.

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