Question Paper

Exam Date & Time: 12-Nov-2018 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTER FOR APPLIED SCIENCES
THIRD SEMESTER BSC APPLIED SCIENCES THEORY EXAMINATION NOVEMBER 2018
MATHEMATICS - III [MA 231]

Marks: 100 **Duration: 180 mins.**

Answer ANY FIVE full Questions.
Missing data, if any, may be suitably assumed

1) (7)

Solve $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$.

1A)

1B) (7)

Solve $x\frac{dy}{dx} + y = x^3y^6$

1C) (6)

Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} 3t & 0 < t \le 2 \\ 6 & 2 < t < 4 \end{cases}, \quad f(t+4) = f(t)$$

2) (7)

Solve $3x(xy-2)dx + (x^3+2y)dy = 0$

2A)

2B) (7)

Solve $\frac{dy}{dx} = x + y$, $y(0) = 1$, by Euler's method, for $x = 1$, taking $h = 0.1$.

2C) (6)

Use convolution theorem and evaluate $L^{-1}\left\{\frac{1}{s(s^2+1)}\right\}$.

3) (7)

Solve $y'' + y' + y = (1 - e^x)^2$.

3A)

3B) (7)

Given $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$. Compute y at $x = 0.1$ by taking $h = 0.05$ using Runge - Kutta method of order four.

3C) (6)

Use Laplace transforms and solve

$$y'''(x) + 2y''(x) - y'(x) - 2y(x) = 0,\ \ y(0) = \ y'(0) = 0, y''(0) = \ 6$$

4) (7)

4A) Solve $(D-2)^2 y = 8(e^{2x} + sin2x + x^2)$ where $D = \frac{d}{dx}$.

4B) (7)

Solve $u_{xx} - 4u_{xy} + 3u_{yy} = 0$ using the transformation $v = x + y,\ \ z = 3x + y$.

4C) (6)

Show that $v(x,y) = -sinxsinhy$ is harmonic. Also find the analytic function $f(z) = u + iv$.

5) (7)

5A) Find $F(t) = L^{-1}\left\{\frac{e^{-3s}}{(s+4)^{\frac{5}{2}}}\right\}$.

5B) (7)

Evaluate: $\oint |z|^2 dz$ around the square with vertices at $(0,0), (1,0), (1,1), (0,1)$.

5C) (6)

Using the method of variation of parameters,

solve $y'' + 4y = \tan 2x$.

6) (7)

6A) Use Laplace transforms and evaluate the integral $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$.

6B) (7)

Solve $x^2y'' - 4xy' + 6y = x^2$.

6C) (6)

Evaluate $\int_C f(z)\,dz$, where $f(z) = \frac{z^2 - z + 1}{z - 1}$ and C is the circle $i)|z| = 1, ii)\ |z| = \frac{1}{2}$.

7) (7)

7A) Solve $(2x+3)^2 \frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$

7B) (7)

Evaluate $\oint \frac{z+2}{z(z-1)} dz$ where C is the circle $i)|z| = \frac{1}{2}, ii)\ |z| = 2$, using Cauchy's residue theorem.

7C) (6)

Find the Laplace transfer of $f(t) = te^{-2t}cos2t$.

8) (7)

8A) Solve $\frac{\delta^2 z}{\delta x^2} - 2\frac{\delta z}{\delta x} + \frac{\delta z}{\delta y} = 0$, using method of separation of variables.

8B) (7)

Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$.

8C) (6)

Find the Taylor's expansion of $f(z) = \frac{1}{(z+1)^2}$ about the point $z = -i$

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