Question Paper

Exam Date & Time: 12-Nov-2018 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
THIRD SEMESTER B.Sc THEORY EXAMINATION NOV . 2018
Mathematics -III [IMA 231 - S2]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.
Missing data, if any, may be suitably assumed

1)

1A) Solve $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0.$ (7)

1B) Using method of separation of variables, solve $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}=0.$ (7)

1C) Express the given function $f(t)=\begin{cases}1 & 0<t\le 1\\ t & 1\le t\le 2\\ t^2 & t\ge 2\end{cases}$ in terms of unit step function and hence find the Laplace transform. (6)

2)

2A) Solve $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0.$ (7)

2B) Solve $\frac{dy}{dx}=\log(x+y),\ y(0)=2$, by Euler's modified method, for $x=0.2$ and $x=0.4$ taking h = 0.2. Carry out two iterations for each step. (7)

2C) Using convolution theorem, evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}.$ (6)

3)

3A) Solve $(D^2-4D+3)y=2xe^{3x}$, where $D=\frac{d}{dx}.$ (7)

3B) Given $\frac{dy}{dx}=x+y,\ y(0)=1.$ Compute y at $x=0.4$ by taking $h=0.2$ using Runge - Kutta method of order four. (7)

3C) Use Laplace transforms and solve $y''+2y'+2y=5\,sint$, $y(0)=y'(0)=0.$ (6)

4)

4A) Solve $(D^4+8D^2+16)y=2\cos^2 x$ where $D=\frac{d}{dx}.$ (7)

4B) (7)

Solve $u_{xy} - 2u_{yy} = 0$ using the transformation $v = x,\ z = x + y$.

4C) Find the orthogonal trajectories of the family of curves $x^3y - xy^3 = c$, where c is a constant. (6)

5) (7)

5A) Find inverse Laplace transform of $\frac{1}{s^2(s+a)^2}$.

5B) Evaluate: $\int_C |z|^2\, dz$ around the square with vertices at (0, 0), (1, 0), (1, 1) and (0, 1). (7)

5C) Using the method of variation of parameters, solve $y'' + 4y = \tan 2x$. (6)

6) (7)

6A) Find the Laplace transform of $f(t) = \frac{2\sin t \sin 5t}{t}$.

6B) Solve $x^2y'' - 3xy' + 5y = x^2 \sin(\log x)$. (7)

6C) Evaluate $\int_C f(z)\, dz$, where $f(z) = \frac{\cos \pi z}{(z^2-1)}$ around a rectangle with vertices $2 \pm i, -2 \pm i$, using Cauchy's integral formula. (6)

7) (7)

7A) Solve the simultaneous differential equations

$\frac{dx}{dt} + 2y = -\sin t, \frac{dy}{dt} - 2x = \cos t$.

7B) State Cauchy's residue theorem. Hence evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle $|z| = 2.5$. (7)

7C) Find Taylor's series expansion of $f(z) = \frac{2z^3+1}{z^2+z}$ about the point $z = i$. (6)

8) (7)

8A) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta}\, d\theta$ around a unit circle.

8B) Solve $x\frac{dy}{dx} + y = x^3y^6$. (7)

8C) (6)

Show that $u(x,y) = \frac{1}{2}\log(x^2 + y^2)$ is harmonic. Also find the analytic function $f(z) = u + iv$.

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