Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

A Constituent Institution of Manipal University

III SEMESTER B.TECH. (ECE/EEE/ICE/BME)

END SEMESTER EXAMINATIONS, NOV. 2018

SUBJECT: ENGINEERING MATHEMATICS-III [MAT 2102]

REVISED CREDIT SYSTEM

(22/11/2018)

Time: 3 Hours MAX. MARKS: 50

Instructions to Candidates:

- Answer **ALL** the questions.

1A.	Find the Fourier series of $f(x) = x(2\pi - x), 0 \le x \le 2\pi, f(x+2\pi) = f(x)$ and hence find the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$.	**4**
1B.	Obtain the half range Fourier cosine series of $f(x) = 1 - \frac{x}{l},\ 0 \le x \le l.$	**3**
1C.	Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & \lvert x \rvert \le a \\ 0, & \lvert x \rvert > a \end{cases}$ and hence evaluate $\int_0^{\infty} \frac{\sin t - t\cos t}{t^3} dt.$	**3**
2A.	Find the Fourier sine and cosine transform of $f(x) = \frac{1}{\sqrt{x}}$.	**4**
2B.	Find the analytic function f(z) = u+iv for which $v = e^{-x}(x\cos y + y\sin y)$	**3**
2C.	If f(z) = u + iv is analytic function of z , show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) u^p = p(p-1) u^{p-2} \lvert f'(z) \rvert^2$.	**3**
3A.	(i)Find all possible expansion of $\frac{z+1}{z^2 - z - 2}$ about z = 0. (ii) Expand $f(z) = e^z$ about z = πi.	**4**
3B.	Evaluate $\oint_C \frac{z}{(z^3 - 5z^2 + 8z - 4)} dz \ \ where\ C : \lvert z - 2 \rvert = 2.$	**3**
3C.	Find the directional derivative of $x^2y^2z^2$ at (1, 1, 1) along the unit normal to the surface $x^2 + y^2 + z^2 = 4$ at $(1, \sqrt{2}, 1)$. In what direction it is maximum? Find the maximum value.	**3**
4A.	Given that $\boldsymbol{F} = (2x + y^2)\boldsymbol{i} + (3y - 4x)\boldsymbol{j}$. Verify Green's theorem for $\oint_C \boldsymbol{F}.\boldsymbol{dr}$ around the triangle formed by the points $(0,0), (2,0)$ and $(2,1)$.	**4**

4B.	Prove that $\vec{F} = \left(y^2 \cos x + z^3\right)\hat{i} + \left(2y \sin x - 4\right)\hat{j} + \left(3xz^2 + 2\right)\hat{k}$ is a conservative force field, find its scalar potential and also find the work done by $\vec{F}$ in moving an object in this field from (0,1,-1) to $\left(\frac{\pi}{2}, -1, 2\right)$.	**3**
4C.	If f(r) is a differentiable function of $r = \lvert\vec{r}\rvert$ then show that $f(r)\vec{r}$ is irrotational. Find f(r) so that $f(r)\vec{r}$ is also solenoidal.	**3**
5A.	Verify Stoke's theorem for $\vec{A} = (2x-y)i - yz^2 j - y^2 zk$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary.	**4**
5B.	Derive the D'Alembert's solution of the one dimensional wave equation subject to the initial conditions u(x,0) = f(x) and $\frac{\partial u}{\partial t}(x,0) = 0.$	**3**
5C.	Assuming the most general solution, solve the one dimensional heat equation $u_t = c^2 u_{xx}$ in a laterally insulated bar of length 10 cms whose ends are kept at zero and the initial temperature is $f(x) = x(10 - x), 0 \le x \le 10.$	**3**