MANIPAL
ACADEMY of HIGHER EDUCATION

Reg. No.										

DEPARTMENT OF SCIENCES, I SEMESTER M.Sc PHYSICS
END SEMESTER EXAMINATIONS, DECEMBER 2018

MATHEMATICAL METHODS OF PHYSICS [CODE- PHY 4101]
(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours | Date: 21-12-2018 | MAX. MARKS: 50

Note: (i) Answer **ALL** questions

(ii) Draw diagrams, and write equations wherever necessary

1A. State and prove Cauchy's integral theorem for simply connected region. Explain how this can be extended to multiply connected region.

1B. Define residue. Using the residue theorem, evaluate, $I = \int_{-\infty}^{+\infty} \frac{dx}{1+x^2}$

1C. Test for analyticity of the function of complex variable. (i) $f(z) = |z|^2$ (ii) $f(z) = z^2$

[4 + 4 + 2]

2A. Divergence of the vector field A in curvilinear coordinates is given by,

$$\nabla \cdot A = \frac{1}{r_1 r_2 r_3}\left[\frac{\partial}{\partial h_1}(A_1 r_2 r_3) + \frac{\partial}{\partial h_2}(A_2 r_1 r_3) + \frac{\partial}{\partial h_3}(A_3 r_1 r_2)\right]$$

Using the above, obtain an expression for Laplacian in curvilinear coordinates.

2B. Define and deduce the properties of unitary and orthogonal matrices.

2C. Find out the eigen values and eigen vectors of the following matrix.

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

[3 + 2 + 5]

3A. Starting from the generating function, arrive at the series form of Bessel function of first kind, $J_n(x)$. Show that $J_{-n}(x) = (-1)^n J_n(x)$

3B. Using Legendre polynomial, obtain an expression for potential at a point due to an electric dipole.

[5 + 5]

4A. Define (i) cyclic group (ii) permutation group

4B. Show that velocity is contravariant vector.

4C. Perform the symmetry transformations of a square and show that the set of all symmetry transformations which leave a physical system invariant form a group.

[2 + 2 + 6]

5A. Represent the function $f(x) = x, \quad -\pi \le x \le \pi$ in the form of a Fourier series, and show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots = \frac{\pi}{4}$. Show the graphical representation of $f(x)$ in $[\pi, \ \pi]$ and periodic expansion outside of $[-\pi, \ \pi]$.

5B. Find the inverse Laplace transform of the function $g(s) = \frac{s}{(s^2+a^2)(s^2+b^2)} \quad a^2 \neq b^2$

[5 + 5]