MANIPAL
ACADEMY of HIGHER EDUCATION

Reg. No.										

DEPARTMENT OF SCIENCES, I/III SEMESTER M.Sc (PHYSICS)
END SEMESTER EXAMINATIONS, NOVEMBER 2018
SUBJECT : Mathematical Methods of Physics [PHY 4101]
(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date: 19-11-2018 MAX. MARKS: 50

Note: (i) Answer **ALL** questions
(ii) Draw diagrams, and write equations wherever necessary

1A. Using the generating function, arrive at the series form of Hermite function, $H_n(x)$ and show that $H_n(-x) = (-1)^n H_n(x)$ **5**

1B. The generating function for Legendre polynomial, $P_n(x)$ and Bessel function, $J_n(x)$ are $g(x,t) = (1-2xt+t^2)^{-\frac{1}{2}}$ and $g(x,t) = e^{\frac{x}{2}\left(t-\frac{1}{t}\right)}$ respectively. Prove the following.

(i) $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$

(ii) $J_{n-1}(x) - J_{n+1}(x) = 2J_n'(x)$

(iii) $P_n(1) = 1$ **(2+2+1)**

2A. State Stokes' theorem. Obtain an expression for curl of a vector field in curvilinear coordinates. **5**

2B. Mention various transformation for diagonalization of matrices. Diagonalize the following real symmetric matrix using congruence transformation.

$$A = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 5 & -1 \\ 3 & -1 & 3 \end{bmatrix}$$ **5**

3A. Define contravariant, covariant tensors. Find the metric for (i) the three-dimensional Euclidean space, and (ii) the surface of a sphere of constant radius 'a' in terms of spherical polar coordinates. **5**

3B. Perform the symmetry transformation of an equilateral triangle and show that the set of all symmetry transformations which leave a physical system invariant forms a group. **5**

4A. State and prove the theorem that goes by the name 'Cauchy integral formula'. **5**

4B. State residue theorem. Using residue theorem, evaluate, $\int_0^\infty \frac{cosx}{x^2+1}dx$ **5**

5A. (i) Find the inverse Laplace transform of $g(s) = \frac{k^2}{s(s^2+k^2)}$. **3**

(ii) Use the theory of Laplace transform to describe simple harmonic motion. **3**

5B. Represent the function, $f(x) = x^2, \quad -\pi < x < \pi$ in the form of Fourier series, and show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. **4**