Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

I SEMESTER M.TECH. (AUTOMOBILE ENGINEERING)

END SEMESTER EXAMINATIONS, NOVEMBER/DECEMBER- 2018

SUBJECT: COMPUTATIONAL METHODS [MAT 5103]

REVISED CREDIT SYSTEM

Time: 3 Hours (01/12/2018) MAX. MARKS: 50

Instructions to Candidates:
- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1A.	Use SOR method with $\omega = 1.25$ to solve $\begin{aligned} 3x_1 - x_2 + x_3 &= -1 \\ -x_1 + 3x_2 - x_3 &= 7 \\ x_1 - x_2 + 3x_3 &= -7 \end{aligned}$. Carryout 2 iterations using $X^{(0)} = (0\ 0\ 0)^T$ and accurate to 4 decimal places.	**3 Marks**
1B.	Solve by Newton-Raphson method: $f(x,y) = x - x^2 - y^2$ and $g(x,y) = y - x^2 + y^2$, starting with initial approximation (0.8, 0.4).	**3 Marks**
1C.	Solve the system of equations using Thomas method: $\begin{bmatrix} 2.04 & -1 & 0 & 0 \\ -1 & 2.04 & -1 & 0 \\ 0 & -1 & 2.04 & -1 \\ 0 & 0 & -1 & 2.04 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x \end{bmatrix} = \begin{bmatrix} 40.8 \\ 0.8 \\ 0.8 \\ 200.8 \end{bmatrix}$	**4 Marks**
2A.	Given $y' = y + e^x$, $y(0) = 0$; find $y(0.4)$ with $h = 0.2$ by Euler's modified method.	**3 Marks**
2B.	Evaluate $y(1.4)$ by Adams-Bashforth method. Given $y' = x^2(1+y)$ and $y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979$	**3 Marks**

2C.	Solve the differential equations $y'' = x(y')^2 - y^2$ for $x = 0.2$ using R K method. Take the initial values are $x = 0, y = 1, y' = 0$ and $h = 0.2.$	**4 Marks**
3A.	Solve $y'' = xy,\ y(0) + y'(0) = 1,\ y(1) = 1$ with $h = 0.5$ by finite difference method.	**3 Marks**
3B.	Solve $u_{tt} = u_{xx},\ 0 < x < 2,\ t \geq 0$. Given that $u(0,t) = u(2,t) = 0,$ $u(x,0) = 0,\ u_t(x,0) = 100(2x - x^2)$. Choosing $h = 0.5,$ compute u for 4 time steps.	**3 Marks**
3C.	Solve $u_{xx} + u_{yy} = -81xy,\ 0 < x < 1,\ 0 < y < 1.$ $Given\ that\ u(x,0) = u(0,y) = 0,\ u(1,y) = u(x,1) = 100, take\ h = \frac{1}{3}.$	**4 Marks**
4A.	Use Galerkin's method to solve the boundary value problem $y'' - y + x = 0,\ 0 < x < 1,\ y(0) = 0, y(1) = 0.$	**3 Marks**
4B.	Apply Rayleigh-Ritz method to solve $y'' = 3x + 4y,\ 0 < x < 1,\ y(0) = 0, y(1) = 1.$ (Use one parameter approximate solution).	**3 Marks**
4C.	Solve the heat equation $32u_t = u_{xx}, 0 \leq x \leq 1, t > 0$ by Schmidt method, subject to the conditions $u(0,t) = 0, u(1,t) = t$ and $u(x,0) = 0$. Take $\lambda = 1/2,\ h = 1/4$ compute u for 4-time steps.	**4 Marks**
5A.	The annual birth and death rates in a country are 11.7% and 10.78% respectively. While the annual immigration and emigration rates are 20.23% and 14.95%. Assuming the rates to be constant over a period of five years. Use differential equation to formulate a model for population change and predict the populations of the next five years, if the current population is 368250.	**3 Marks**
5B.	Discuss the 12 point procedure and classification of mathematical modelling.	**3 Marks**
5C.	Solve $u_t = u_{xx} + u_{yy}$ satisfying the initial condition: $u(x,y,0) = \sin 2\pi x \sin 2\pi y,\ 0 \leq x, y \leq 1$ and conditions $u(x,y,t) = 0,\ t > 0$ on the boundaries. Using ADE method, obtain the solution upto two time levels with $h = 1/3,\ \alpha = 1/8.$	**4 Marks**