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I SEMESTER M.TECH. (CHEMICAL & BIOTECHNOLOGY)
END SEMESTER MAKE-UP EXAMINATION, January 2019

SUBJECT: MATHEMATICAL & NUMERICAL TECHNIQUES FOR CHEMICAL AND BIOTECHNOLOGY ENGINEERING [CODE- MAT-5102]
(REVISED CREDIT SYSTEM)

| Time: 3 Hours | Time: 2pm to 5pm | MAX. MARKS: 50 |

Answer **ALL** questions

1A. Using Given's method reduce the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \\ 3 & 2 & 3 \end{bmatrix}$ to tridiagonal form.

1B. Prove that $\int_{-1}^{1} \dfrac{T_m(x)T_n(x)}{\sqrt{1-x^2}}\, dx = \begin{cases} 0, & m \neq n \\ \dfrac{\pi}{2}, & m = n \neq 0 \\ \pi, & m = n = 0 \end{cases}$.

1C. Apply Newton-Raphson method to determine a root of the equation $x^4 - x - 10 = 0$ Carryout three iterations. **(4+3+3)**

2A. Given $y'' = xy'^2 - y^2$, $y(0) = 1$, $y'(0) = 0$. Find y and y' at x = 0.2 by Runge Kutta method of order 4.

2B. In the table below the value of y are consecutive terms of a series of which 23.6 is the sixth term. Find the first and tenth terms of the series.

x	3	4	5	6	7	8	9
y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

2C. Construct a least square approximation of second degree for f(x) = sinx, $x \in \left[0, \dfrac{\pi}{2}\right]$.
(4+3+3)

3A. Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence deduce that

$$\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$$

3B. Solve the following equation by Gauss – Seidal method, carry out four iterations.
$27x + 6y - z = 85$; $x + y + 54z = 110$; $6x + 15y + 2z = 72$

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3C.   Solve: $\dfrac{\partial^2 u}{\partial x^2} = +\dfrac{\partial^2 u}{\partial y^2} = -10(x^2 + y^2 + 10),\ \ 0 \le x \le 3,\ \ 0 \le y \le 3,\ h = 1$ **(4+3+3)**

4A   Using the finite difference method find y(o.25), y(0.5) and y(0.75) satisfying the differential

equation $\dfrac{d^2 y}{dx^2} + y = x$ subject to the boundary conditions y(0) = 0, y(1) = 2.

4B   Find the largest eigen value and eigen vector of the matrix A= $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. Start

with $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ and carry out four iterations.

4C   Fit a curve $y = ae^{bx}$ to the following data.

x	0	1	2	3
y	1.05	2.10	3.85	8.30

     (4+3+3)

5A.   Determine the coefficients of the approximate solution $w(x) = a_1\left(1 - x^2\right) + a_2 x^2 \left(1 - x^2\right)$

for the boundary value problem $y'' + \left(1 + x^2\right) y + 1 = 0,\ y(\pm 1) = 0$

by using Galerkin method.

5B   Using suitable interpolation formula find f (2) from the following   table.

x	-4	-1	0	2	5
y	1245	33	5	9	1335

 5C   Solve   the following system of equations using LU decomposition method.

$3x + 2y + 7z = 4;\ \ 2x + 3y + z = 5;\ \ \ 3x + 4y + z = 7.$ **(4+3+3)**