Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

I SEMESTER M.TECH. (CHEMICAL & BIOTECHNOLOGY)
END SEMESTER EXAMINATIONS, DECEMBER 2018

SUBJECT: MATHEMATICAL & NUMERICAL TECHNIQUES FOR CHEMICAL AND BIOTECHNOLOGY ENGINEERING [CODE- 5102]
(REVISED CREDIT SYSTEM)

Time: 3 Hours Date:01-12-2018 MAX. MARKS: 50

Answer **ALL** questions

1A. Using Jacobi's method find all the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$.

1B. Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$, m $\neq$ n.

1C Apply Newton-Raphson method to determine a root of the equation cosx-xe^x = 0. Carryout three iterations. **(4+3+3)**

2A. Given $y'' + xy' + y = 0,\ y(0) = 1,\ y'(0) = 0$. Find y and y' at x = 0.2 by Runge Kutta method of order 4.

2B. The table gives the distance in nautical miles of the visible horizon for the given heights in feet above the earth's surface:

X height	100	150	200	250	300	350	400
Y distance	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when x = 218ft and 410ft.

2C. Using Chebyshev's polynomials obtain the least square approximation of second degree for f(x) = x^4, $x \in [-1,1]$ **(4+3+3)**

3A Find the Fourier transform of $f(x)=\begin{cases} a^2 - x^2. & |x| < a \\ 0, & |x| > a \end{cases}$ and hence deduce that

$$\int_0^\infty \frac{(\text{sint} - t\cos t)^2}{t^6} dt = \frac{\pi}{15}$$

3B Solve the following equation by Gauss – Seidal method, carry out four iterations.

$-2x_2 - x_3 - x_4 + 10x_1 = 3;\ -2x_1 - x_3 - x_4 + 10x_2 = 15;$

$-x_1 + 10x_3 - 2x_4 - x_2 = 27;\ x_1 - 2x_3 + 10x_4 - x_2 = -9$

3C. Solve: $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$, $0 \le x \le 5,\ t>0$ with initial conditions

$$u(x,0)=\begin{cases} 20x, & 0 \le x < 1 \\ 5(5-x), & 1 \le x \le 5 \end{cases} \quad \text{and} \quad \frac{\partial u}{\partial t}(x,0)=0$$ and boundary conditions

u (0, t) = u (5, t) =0. Find u(x,t) for four time steps. Take h = 1. **(4+3+3)**

4A The deflection of a beam is governed by the equation $\frac{d^4y}{dx^4}+81y = \varphi(x)$ where $\varphi(x)$ is given by the table below and the boundary condition $y'(0) = y''(1) = y'''(1) = 0$, y (0) =0. Evaluate the deflection at the pivotal points of the beam using three subintervals.

x	1/3	2/3	1
Φ(x)	81	162	243

4B Find the largest eigen value and eigen vector of the matrix A= $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{bmatrix}$. Start with $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$ and carry out four iterations.

4C Find y at x= 3.75 by fitting a power curve y = ax^b to the following data.

x	1	2	3	4	5	6
y	2.98	4.26	5.21	6.10	6.80	7.50

(4+3+3)

5A. Determine the coefficients of the approximate solution $w(x)=a_1(1-x^2)+a_2x^2(1-x^2)$ for the boundary value problem $y''+(1+x^2)y+1=0,\ y(\pm 1)=0$ by using (a) Partition method, (b) Collocation method.

5B Using suitable interpolation formula find f (15) from the following table.

x	4	5	7	10	11	13
y	48	100	294	900	1210	2028

5C. Using the Gauss-Jordan method, find the inverse of the matrix A= $\begin{bmatrix} 1 & 3 & 3 & 2 \\ 1 & 4 & 3 & 4 \\ 1 & 3 & 4 & 5 \\ 2 & 5 & 3 & 2 \end{bmatrix}$ **(4+3+3)**