MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

A Constituent Institution of Manipal University

FIRST SEMESTER M.TECH (ENVIRONMENTAL ENGINEERING) END SEMESTER

EXAMINATIONS, NOVEMBER 2018

SUBJECT: COMPUTATIONAL METHODS & OPTIMIZATION TECHNIQUES
[MAT -5106]
REVISED CREDIT SYSTEM
(01/12/2018)

Time: 3 Hours   MAX. MARKS: 50

Instructions to Candidates:

❖ Answer **ALL** questions.
❖ All questions carry equal marks.

1A.	Find the minimum value of $f(x) = \frac{x}{\log x}$ by taking $x^{(0)} = 2.5$, step size 0.1 using quadratic interpolation method.	**4**
1B.	Solve: $y'' + x^4 y = 0$, $y(0) = 4$, $y(1) = 1$, $h = 0.25$.	**3**
1C.	In a normal distribution, 7% of the items are under 35 and 89% are under 63. Find the mean and variance of the distribution.	**3**
2A.	Using Simplex method, solve the following LPP, Maximize $z = 3x_1 + 4x_2$ subject to $4x_1 + 2x_2 \le 80, 2x_1 + 5x_2 \le 180$, $x_1, x_2 \ge 0$.	**4**
2B.	It is suspected that a patient has one of the diseases A_1, A_2, A_3. Suppose that the population percentage suffering from these illness are in the ratio 2:1:1. The patient is given a test which turns out to be positive 25% of the cases of A_1, 50% of the cases of A_2 and 90% of A_3. Given that out of three tests taken by the patient two are positive, find the probability for each of the 3 illness.	**3**
2C.	Suppose that the two-dimensional random variable (X, Y) has pdf given by $f(x, y) = \begin{cases} 2(x + y), & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$ then find ρ_{xy}.	**3**

3A.	The following figures show that the distribution of digits in number chosen at random from a telephone directory. Test whether the digits occur equally frequently at 0.05 significance level.	4

Digit	0	1	2	3	4	5	6	7	8	9	total
Frequency	1026	1107	997	966	1075	933	1107	972	964	853	10000

3B.	Find the real root of the equations $x^2 - y^2 - 1 = 0$ and $x^2 + y^2 - 9 = 0$ near $x_0 = y_0 = 2.1$, by Newton – Raphson's method. Carry out 2 iterations.	3

3C.	Let $\overline{X}$ denote the mean of a random sample of size 100 from the distribution $\chi^2(50)$. Compute an approximate value of $\Pr\{49 < \overline{X} < 51\}$.	3

4A.	Solve : $u_t = u_{xx}$, $0 < x < 1, t > 0$, $u(x, 0) = 100\,(x - x^2)$, $u(0, t) = u(1, t) = 0$. Taking $h = 0.25$, compute u for 4 time steps.	4

4B.	Fit the curve $y = ae^{bx}$ to the following data.	3

x	2	3	4	5	6
y	144	172	207	249	299

4C.	Minimize the function $f(x) = x(x - 4)$ in the interval [1, 3] by the Fibonacci method. Take n = 3.	3

5A.	Using dual simplex method, minimize $z = 2x_1 + 2x_2 + 4x_3$ subject to $2x_1 + 3x_2 + 5x_3 \geq 2$, $3x_1 + x_2 + 7x_3 \leq 3$, $x_1 + 4x_2 + 6x_3 \leq 5$, $x_1, x_2, x_3 \geq 0$	4

5B.	Find the m.g.f of the random variable X which is uniformly distributed over (-a, a). Evaluate $E(x^{2n})$.	3

5C.	Find the Mean, Median and Mode for the following data	3

Class	1-10	11-20	21-30	31-40	41-50	51-60
Frequency	3	16	26	31	16	8