Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

I SEMESTER M.TECH. (EC/BM)

END SEMESTER EXAMINATIONS NOVEMBER/DECEMBER 2018

SUBJECT: PROBABILITY, RANDOM VARIABLES AND STOCHASTIC PROCESSES [MAT 5104]

Date of Exam: **01/12/2018** Time of Exam: **02.00 PM – 05.00 PM** Max. Marks: **50**

Instructions to Candidates:

- Answer ALL the questions. Use of STATISTICAL TABLES is permitted

1A. A watchman enters a cave with two flashlights, one that contains three batteries in series (one on top of the other) and another that contains two batteries in series. Assume that all batteries are independent and that each will work with probability 0.9. Find the probability that the watchman will have some means of illumination.

1B. Let X be a random variable whose probability mass function is $p_X(x)=\begin{cases}\alpha/x, x=1,2,3,4\\ 0, otherwise\end{cases}$

(i) What is the value of α?

(ii) Find $P[X \text{ is odd}]$

(iii) Also determine $P[X > 2]$

1C. From the past records it has been observed that on Swami Vivekananda's birthday (January 12), the probability that it rains in Kolkata is 0.75. Two television stations are noted for their weather forecasting abilities. The first, which is correct nine times out of ten, says that it will rain on the upcoming birthday; the second, which is correct eleven times out of 12, says that it will not rain. What is the probability that it will rain on the upcoming birthday of Vivekananda? **(3+3+4)**

2A A discrete random variable X takes the value 1 if the number 6 appears on a single throw of a fair die and takes the value 0 otherwise. Find its probability generating function.

2B. The age of a randomly selected person in a certain population is a normally distributed random variable X. Furthermore, it is known that $P[X \le 40] = 0.5$ and $P[X \le 30] = 0.25$. Find the mean and standard deviation of X.

2C The joint probability density function of two random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases}\frac{1}{2}, -1 \le x \le y \le 1,\\ 0, otherwise\end{cases}$$

Find $f_{X|Y}(x|y)$, $f_{Y|X}(y|x)$, $E[X|y]$ and $E[Y|x]$ **(3+3+4)**

3A One observation X is taken from $N(0, \sigma^2)$ population. Find the maximum likelihood estimator of σ.

3B Let X be an exponentially distributed random variable with mean $1/\theta$. Find the probability density function and the cumulative distribution function of $Y = X^2$

3C. The following experiment was performed to assess the relative effects of four toxins on the liver of a certain species of trout. The data are the amounts of deterioration (in standard units) of the liver in each sacrificed fish. Test whether the data gives evidence that toxins produce different effects.
Table values of F(3,15) and F(15,18) are 3.29 and 3.16 respectively, at 5% level of significance. **(3+3+4)**

Toxin 1	28	23	14	27		
Toxin 2	33	36	34	29	31	34
Toxin 3	18	21	20	22	24	
Toxin 4	11	14	11	16		

4A Suppose that there are five types of breakfast cereal, which we call A, B, C, D and E. Customers tend to stick to the same brand. Those who choose type A, choose it the next time around with probability 0.8; those who choose type B, choose it the next time with probability 0.9. The probabilities for types C, D and E are given by 0.7, 0.8 and 0.6, respectively. When customers do change brand, they choose one of the other four equally probably. Explain how this may be modelled by a Markov chain and give the transition probability matrix.

4B Let C be a non-closed communicating class. Show that no state in C can be recurrent.

4C. Consider a discrete-time Markov chain consisting of three states 0, 1 and 2 and whose transition probability matrix is given by $\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/4 & 1/2 & 1/4 \\ 1/6 & 1/3 & 1/2 \end{bmatrix}$. Show that this chain has a unique stationary distribution π and find π. **(3+3+4)**

5A Let $W_q(t) = P[T_q \le t]$ be the waiting time distribution of an M/M/1 queue. Derive an expression for $W_q(t)$ in terms of the probability of an arriving customer finding *n* customers already present.

5B Derive the transient state distribution for the M/M/1 model.

5C Customers arrive at a car wash service at a rate of one every 20 minutes and the average time it takes for a car to proceed through their single wash station is 8 minutes. Answer the following questions under the assumption of Poisson arrivals and exponential service.
(i) What is the probability that an arriving customer will have to wait?
(ii) What is the average number of cars waiting to begin their wash?
(iii) What is the probability that there are more than five cars altogether?
(iv) What is the probability that a customer will spend more than 12 minutes actually waiting before his car begins to be washed? **(3+3+4)**

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