Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

FIRST SEMESTER M.TECH. (E & C) DEGREE END SEMESTER EXAMINATION
DECEMBER 2018/JANUARY 2019
SUBJECT: DETECTION & ESTIMATION THEORY (ECE - 5104)

TIME: 3 HOURS **MAX. MARKS: 50**

Instructions to candidates
- Answer **ALL** questions.
- Missing data may be suitably assumed.

1A. Consider the hypothesis testing problem in which

$$f_{Y/H_0}(y/H_0) = e^{-y} \; for \; y > 0 \; and \; f_{Y/H_1}(y/H_1) = \begin{cases} 1, & 0 \le y \le 1 \\ 0, & otherwise \end{cases}$$

a. Setup the likelihood ratio test and determine the decision regions.
b. Find the minimum probability of error when $P_0 = ½$.

1B. What do you mean by a consistent estimate? State the consistency theorem of MLEs.

1C. Discuss Neyman Pearson criterion for hypothesis testing.

(5+3+2)

2A. Consider a received signal $Y(t) = \sqrt{(2/T)}\cos(2\pi f_c t) + W(t), 0 < t < T_s$, where $T_s = n/f_c$. Given that T is an unknown positive constatnt and W(t) is AWGN, describe a receiver schematic to estimate $1/\sqrt{T}$.

2B. Given the conditional density functions

$$f_{Y/_{H_1}}(^{y}/_{H_1}) = \frac{1}{2\sqrt{2\pi}} exp\left(\frac{-(y-4)^2}{8}\right)$$
$$f_{Y/_{H_0}}(^{y}/_{H_0}) = \frac{1}{2\sqrt{2\pi}} exp\left(\frac{-y^2}{2}\right)$$

Find the decision regions using minimax rule. Assume uniform cost.

(6+4)

3A. The observations under each hypothesis are given by

$H_1 : Y_k = m + N_k$

$H_0 : Y_k = N_k$, k=1,2,….K

Where N_k denotes statistically independent Gaussian random variables of zero mean and variance σ^2 each and m is unknown. Obtain the generalized LRT for the given case and derive an expression for the probability of false alarm P_F.

3B. Show that the MMSE, MMAE and MAP estimates of a real random parameter are respectively the mean, median and mode of conditional probability distribution of the parameter.

3C. Prove Parseval's theorem for a deterministic signal.

(5+3+2)

4A. Consider the binary detection problem
$H_1 : Y(t) = s_1(t) + W(t), \quad 0 \le t \le 2$
$H_0 : Y(t) = s_0(t) + W(t), \quad 0 \le t \le 2$
Where $s_1(t) = -s_0(t) = e^{-t}$, and W(t) is an additive white Gaussian Noise with zero mean and covariance function $C_{ww}(t,u) = (N_0/2)\,\delta(t-u)$.
Draw the optimum receiver, assuming minimum probability of error criterion.
Calculate the minimum probability of error.

4B. Let $Y_1, Y_2, \ldots, Y_K$ be K independent observed random variables, each having a Poisson distri given by

$$f_{Y_k/\Theta}\left({}^{y_k}\!/_{\theta}\right) = e^{-\theta}\frac{\theta^{y_k}}{y_k!}, \quad y_k \ge 0, k = 1,2,\ldots,K$$

The parameter θ is unknown.
a. Obtain the ML estimate of θ.
b. Verify that the estimator is unbiased and determine the lower bound.

(6+4)

5A. Consider the signals shown in **Figure 5A**. Use the Gram-Schmidt procedure to determine the orthonormal basis functions for $s_k(t), k = 1,2,3,4$.

5B. State and prove the orthogonality principle with respect to linear transformations.

5C. Explain the Karhunen Loeve series expansion of a random process.

(5+3+2)

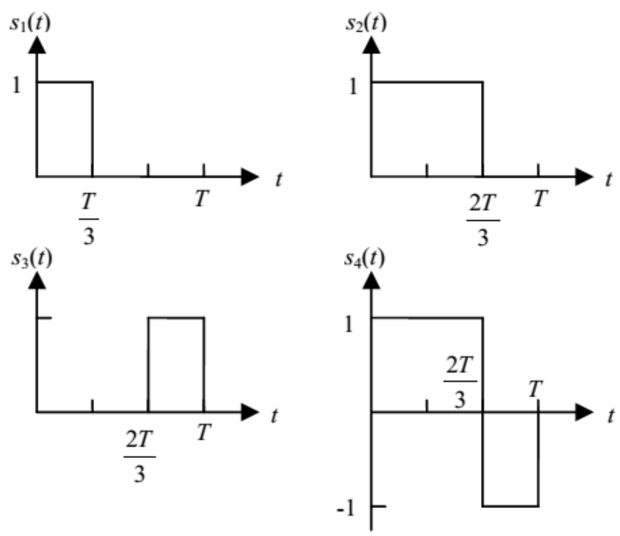

Figure 5A