Reg.No.

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

SEMESTER M.TECH.END SEMESTER EXAMINATIONS- DECEMBER 2018

SUBJECT: FEM FOR THERMAL ENGINEERING [MME 5142]

Time: 3 Hours. MAX.MARKS: 50

Instructions to Candidates:

- Answer **ALL** questions.
- Missing data, if any, may be suitably assumed.

Q.1A Starting from an assumed thermal functional for a slender one dimensional fin and undergoing steady state heat transfer, apply the Variational Finite Element Formulation to obtain the Thermal Conductance Matrix and corresponding Load matrices. Assume a uniform heat generation of q_g W/m^3. (06)

$$[\mathrm{K}]\{\mathrm{T}\}=\{\mathrm{Q}\} \quad where,$$

$$[\mathrm{K}]=\iiint_V [\mathrm{B}]^{\mathrm{T}}[\mathrm{D}][\mathrm{B}]\,dV+\iint_{S_2} \mathrm{h}[\mathrm{N}]^{\mathrm{T}}[\mathrm{N}]\,ds$$

$$\{Q\}=\iiint_V \dot{q}_g [\mathrm{N}]^{\mathrm{T}} dV+\iint_{S_3} \mathrm{hT}_\infty [\mathrm{N}]^{\mathrm{T}} ds$$

Q.1B Determine the Shape functions for the four noded Isoparametric rectangular element using Lagrange's Interpolation Formula. Prove that the general four noded quadrilateral element shown in Fig 1 can be mapped exactly to an Isoparametric rectangular element (for any one side) (04)

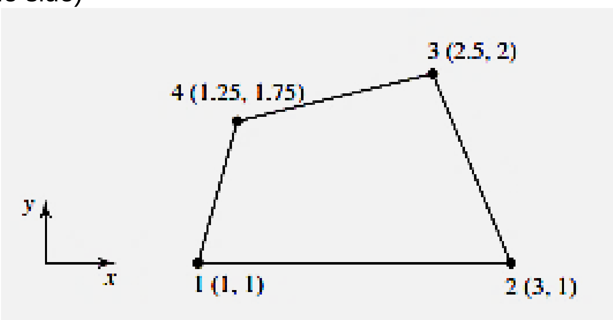

Fig 1

Q.2A Apply Galerkin's Weighted Residual Method for the finite element formulation for a general **3-D cartesian steady state heat transfer system without convective loads** to prove with usual notations, that (07)

$$\left[\iiint\left(k_x\left[\frac{\partial N}{\partial x}\right]^T\left[\frac{\partial N}{\partial x}\right]+k_y\left[\frac{\partial N}{\partial y}\right]^T\left[\frac{\partial N}{\partial y}\right]+k_z\left[\frac{\partial N}{\partial z}\right]^T\left[\frac{\partial N}{\partial z}\right]\right)dV\right]\{T\}=$$

$$\left\{\iiint \dot{q}_g [N]^T dV\right\}+\left\{\iint\left(\dot{q}_x\hat{n}_x+\dot{q}_x\hat{n}_x+\dot{q}_x\hat{n}_x\right)[N]^T dA\right\}$$

Q.2B What is meant by an Isoparametric element? Enumerate the salient differences and relative advantages of Sub-Parametric, Isoparametric and Super-Parametric finite elements by means of neat illustrative examples. (03)

Q.3A For the following thermal systems deduce their GDE using Lagrange-Euler Equations. Assume suitable functional in each case: (03)

1. One dimensional steady state heat transfer with uniform heat generation
2. One dimensional steady state heat transfer in a slender fin with convective end face.
3 Two Dimensional steady state heat transfer with surface

Q.3B Apply Galerkin's weighted residual formulations to obtain the stiffness conductance analogous matrix as well as the thermal load matrix for an **axisymmetric triangular thermal element.** (07)

$$[\mathbf{K}] = \frac{2\pi\overline{r}k_r}{4A}\begin{bmatrix} b_i^2 & b_ib_j & b_ib_k \\ b_ib_j & b_j^2 & b_jb_k \\ b_ib_k & b_jb_k & b_k^2 \end{bmatrix} + \frac{2\pi\overline{r}k_z}{4A}\begin{bmatrix} c_i^2 & c_ic_j & c_ic_k \\ c_ic_j & c_j^2 & c_jc_k \\ c_ic_k & c_jc_k & c_k^2 \end{bmatrix}$$

$$+\frac{2\pi hl_{ij}}{12}\begin{bmatrix} 3r_i + r_j & r_i + r_j & 0.0 \\ r_i + r_j & r_i + 3r_j & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$\{\mathbf{f}\} = \frac{2\pi GA}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{Bmatrix} r_i \\ r_j \\ r_k \end{Bmatrix} - \frac{2\pi q l_{jk}}{6}\begin{Bmatrix} 0 \\ 2r_j + r_k \\ r_j + 2r_k \end{Bmatrix} + \frac{2\pi hT_a l_{ij}}{6}\begin{Bmatrix} 2r_i + r_j \\ r_i + 2r_j \\ 0 \end{Bmatrix}$$

Q.4A Determine the nodal Conductance Matrix and Thermal Load vector for a three noded linear triangular thermal element as given in **Fig 2** below: Given k = 20W/mm.K; thickness, t = 10 mm, All dimensions in mm (06)

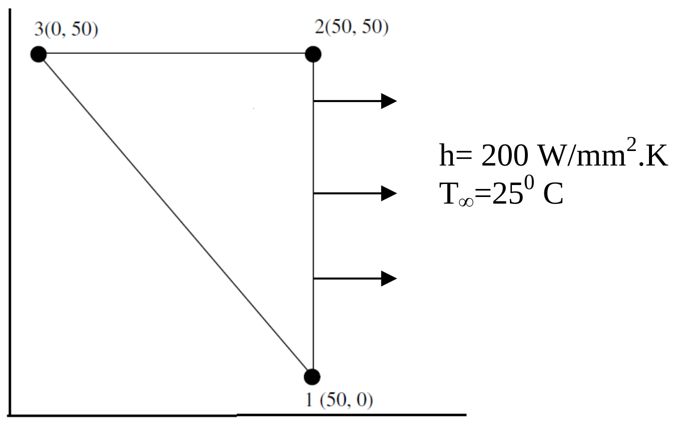

Fig 2

Q.4B Use Serendipity method to determine all the Shape Functions for a Triangular area element with mid-side nodes (04)

Q. 5A Derive two point sampling Gaussian Quadrature Formula. Use the same to find the value of the following Integral: Verify the same with the exact analytical answer. (04)

$$\int_2^3 (2x^2 - 2e^x + 1)dx$$

Q.5B Determine the nodal temperatures for radial heat transfer in a **hollow spherical vessel (FIG.3)** using Discrete finite element analysis. (06)

K_1=25 W/m.K
K_2=10 W/m.K
K_3=0.1 W/m.K
r_1=0.05 m
r_2=0.2 m
r_3=0.3 m
r_4=0.3 m
h_i=45 W/m^2.K
T_i=315^{0}C
T_4=30^{0}C

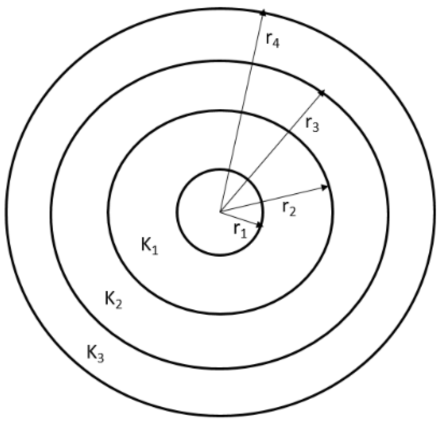

FIG.3