MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

FIRST SEMESTER M.TECH.END SEMESTER MAKE-UP EXAMINATIONS

DECEMBER 2018

SUBJECT: FEM FOR THERMAL ENGINEERING [MME 5142]

Time: 3 Hours. **MAX.MARKS: 50**

Instructions to Candidates:

- Answer **ANY FIVE FULL** questions.
- Missing data, if any, may be suitably assumed.

Q.1A Explain what is meant by Area Coordinates with neat sketches (03)

Q.1B Determine the nodal Conductance Matrix and Thermal Load vector assuming convective heat transfer from both lateral surfaces for a three noded Linear Triangular Thermal element as given shown in **Fig 1** below, (07)

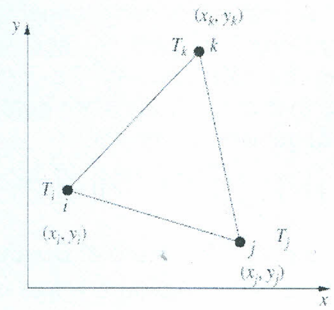
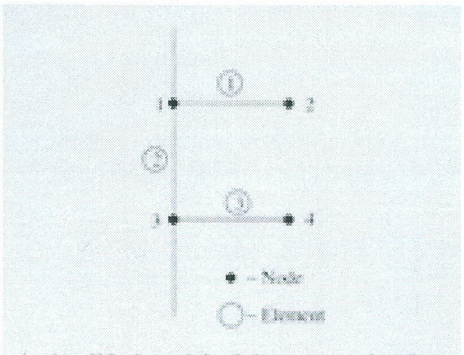


Fig. 1

Q.2A Use Serendipity method to find all the shape functions for an Isoparametric Quadrilateral parent element with mid side nodes and having a centroidal node as well. (04)

Q.2B For the simplified uniaxial two noded fin element having conductive –convective heat transfer, using discrete system analysis, show (with usual notations) that, (06)

$$\begin{bmatrix} \frac{kA}{L} + \frac{hPL}{4} & -\frac{kA}{L} + \frac{hPL}{4} \\ -\frac{kA}{L} + \frac{hPL}{4} & \frac{kA}{L} + \frac{hPL}{4} \end{bmatrix} \begin{Bmatrix} T_i \\ T_j \end{Bmatrix} = \begin{Bmatrix} Q_i + \frac{hPL}{2} T_a \\ -Q_j + \frac{hPL}{2} T_a \end{Bmatrix}$$



A simplified model of the rectangular fins

Q.3A An incompressible fluid flows through a finite element pipe network as shown in Fig. 2. If $0.1\ m^3/s$ of fluid enters the pipe, calculate the nodal pressure and volumetric flow in each pipe. Given $\mu = 0.01\ N.s/m^2$ (05)

Element number	Nodes	Diameter, D (cm)	Length, L (m)
1	1,2	5	25
2	2,3	5	25
2	1,4	5	25
4	4,3	5	25
5	2,4	10	90

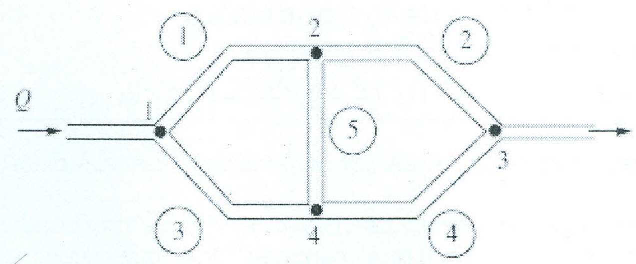


Fig. 2

Q.3B Derive the shape functions for a heat conducting Isoparametric bar element (length, L) having three nodes of constant cross section (A) and having thermal conductivity K, using Lagrangian Interpolation formula. Deduce its thermal conductance using the following formula and assuming that only steady state heat transfer occurs through it with no convective loads and no heat generation as well. (05)

$$[K] = \iiint_V [B]^T [D][B]\, dV$$

Q.4A Use the Variational formulation to obtain the thermal Conductance and load matrices for a slender fin with the open end insulated. Assume a two noded linear steady state heat transfer element. The Governing Differential Equation (with usual notations), is given by, (07)

$$K\frac{d^2T}{dx^2} - \left(\frac{P}{A}\right)h(T - T_\infty) = 0$$

Q.4B A one dimensional quadratic element is used to approximate the temperature distribution in a long fin. The solution gives the temperature as 100^0C ,90^0C and 80^0C. at distances of 100 mm, 150mm, 200 mm respectively from the origin. Compute the temperature and its gradient at a location of 120 mm from the origin. (03)

Q. 5A Derive two point sampling Gaussian Quadrature Formula. Use the same to find the value of the following Integral: Verify the same with the exact analytical answer. (03)

$$\int_3^8 (5x^3 - 2x^2 + 1)dx$$

Q.5B Derive the transient one dimensional heat conduction with usual notation , given by the following thermal equilibrium finite element equation, (07)

$$\frac{\rho c_p l A}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \frac{\partial T_i}{\partial t} \\ \frac{\partial T_j}{\partial t} \end{Bmatrix} + \left(\frac{Ak_x}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hPl}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\right)\begin{Bmatrix} T_i \\ T_j \end{Bmatrix}$$

$$= \frac{GAl}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} - \frac{qPl}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + \frac{hT_a Pl}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$