Reg. No. | | | | | | | | |

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent Institution of MAHE, Manipal)

VII SEMESTER B.TECH (ELECTRICAL & ELECTRONICS ENGINEERING)
END SEMESTER EXAMINATIONS, NOVEMBER 2018
SUBJECT: LINEAR ALGEBRA AND ITS APPLICATIONS TO ELECTRICAL ENGINEERING [ELE 4030]

REVISED CREDIT SYSTEM

Time: 3 Hours **Date: 24 November 2018** **Max. Marks: 50**

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

1A. Define norm of a matrix with relevant axioms. Find the L_0, L_1, L_∞, and $L_{Frobenius}$ norms of the following matrix.

$$B = \begin{pmatrix} 1 & 4 & 0 \\ 4 & 12 & 4 \\ 0 & 4 & 0 \end{pmatrix}.$$

(03)

1B. Consider the linearly independent vectors $u, v \in \mathcal{R}^n$.

(i) State and prove the triangle inequality for u and v.

(ii) State and prove the Cauchy-Schwarz inequality for u and v.

Is it possible to achieve the equality criterion in both? If so, how? ***(04)***

1C. Explain unit L_p ball with relevant contours/shapes drawn on Euclidean plane. ***(03)***

2A. Given the linear system $Ax = b$.

What are well defined, under-defined, and over-defined systems? What is the condition for the system to be consistent? ***(02)***

2B. Using Gauss-Jordan technique find the inverse of the following matrix:

$$A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

(04)

2C. Consider a symmetric matrix $A = \begin{pmatrix} 1 & 4 & 0 \\ 4 & 12 & 4 \\ 0 & 4 & 0 \end{pmatrix}$. Find its LDU factorization and comment on the relation between U and L matrices. ***(04)***

3A. Let A be a $(n \times n)$ symmetric matrix and S be the set of all $(n \times n)$ symmetric matrices. Show that S is a subspace of $M_{3\times3}$, the vector space of (3×3) matrices. ***(02)***

3B. What is the dimension of a subspace? Explain rank and nullity in terms of dimension of a respective subspace. ***(02)***

3C. Describe the column space and null space of A and the complete solution to $Ax = b$. Given:

$$A = \begin{pmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{pmatrix}; \text{ and } b = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}.$$

(04)

3D. State and explain any four properties of determinants. ***(02)***

4A. Derive expressions for least squares-based line fitting technique for sparsely spaced data points. ***(04)***

4B. Find q_1, $q_{2,}$, and q_3 (orthogonal vectors) as combinations of a, b, and c (independent vectors) in A. Then write A as QR.

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{pmatrix}$$

(04)

4C. Prove or disprove the following statements:

(i) Orthogonal transformation of a vector preserves its length

(ii) Row space is orthogonal to null space ***(02)***

5A. Find the possible eigen values and eigen vectors of a matrix obtained by AA^T. Given $A = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ ***(04)***

5B. Prove or disprove that the symmetric matrices have real eigenvalues and orthogonal eigenvectors ***(02)***

5C. Given $A = \begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$. Find A^5 using eigen value decomposition/diagonalization of A. ***(04)***