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**MANIPAL INSTITUTE OF TECHNOLOGY**
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**VII SEMESTER B. TECH (MECHANICAL/IP ENGG.) END SEMESTER MAKEUP EXAMINATIONS, DECEMBER 2018**

SUBJECT: FINITE ELEMENT METHODS [MME 4102]

**REVISED CREDIT SYSTEM**

Time: 3 Hours MAX. MARKS: 50

**Instructions to Candidates:**

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

**1A.** Fig 1a shows the cross section details of a bridge and the average static load acting on it. Span length of the bridge is larger compared to cross section shown. Explain the following steps involved in solving this problem using FEM. **03**

i) Selection of element type
ii) Discretization
iii) Applying boundary conditions –loads and supports

Justify all your selections.

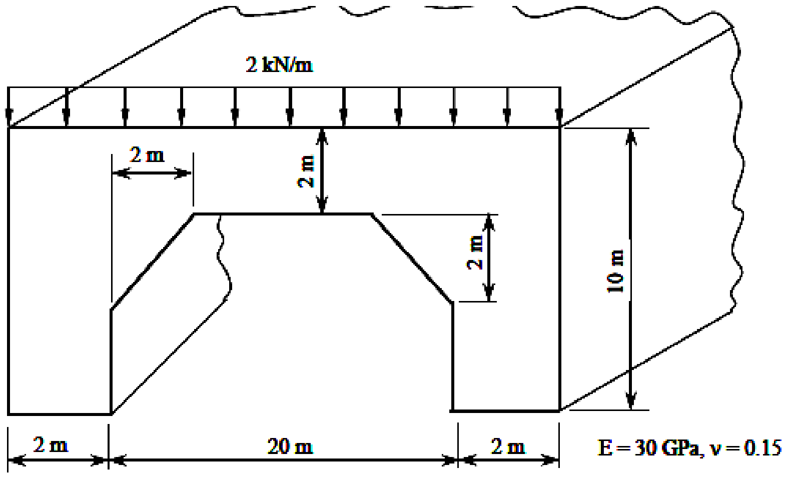



**Fig 1a**

**1B.** Derive the shape functions for 1D three noded horizontal element using Lagrange polynomial **02**

**1C.** Write short notes on **05**

i) Convergence and Methods of obtaining convergence
ii) Eigen value
iii) Minimum potential energy principle
iv) Jacobian Matrix
v) Symmetry Condition

**2A.** Write short notes on following Element types **03**

(i) Bar element
(ii) Beam Element
(iii) CST Element

**2B.** For the structural configuration shown in Fig 2b, use FEM to evaluate the displacements, if any, at the supports. The support at which the 100 kN force is acting, has a vertical downward settlement of 5 mm. Use minimum number of elements. **04**

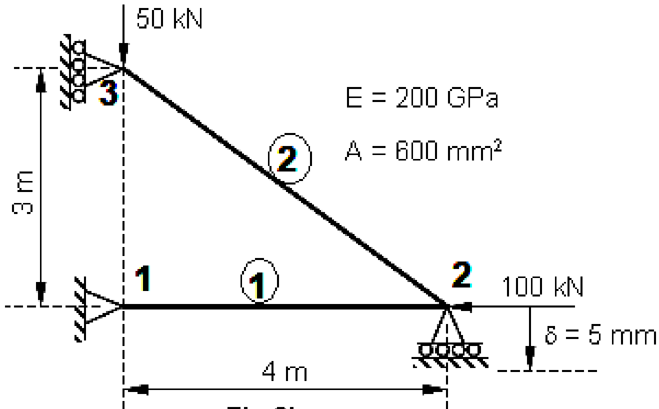



**Fig 2b**

**2C.** Consider the bar shown in Fig 2c. There is an increase in temperature by 50°C. Apply FEM to determine the nodal displacements using one element in the steel part of the bar. Use the data given in table 1. **03**

**Table 1**

| Material | Area of cross section ($mm^2$) | Young's Modulus (GPa) | Coefficient of thermal expansion ($\times 10^{-6}/^\circ C$) | Body force ($N/mm^3$) | Traction force (N/mm) | Change in temperature (°C) |
|---|---|---|---|---|---|---|
| Brass | 250 | 105 | 19 | Nil | 5 | 50 |
| Steel | 400 | 200 | 12 | 0.01 | Nil | |

All dimensions in mm

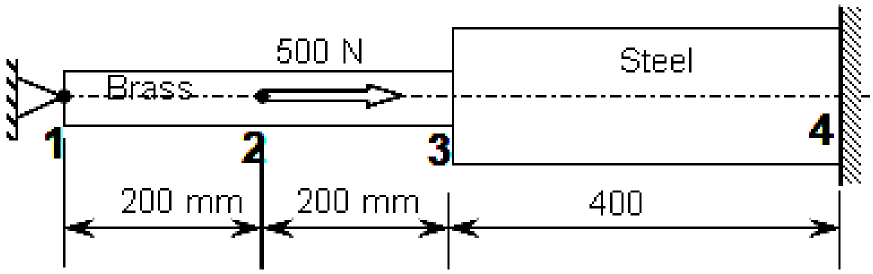
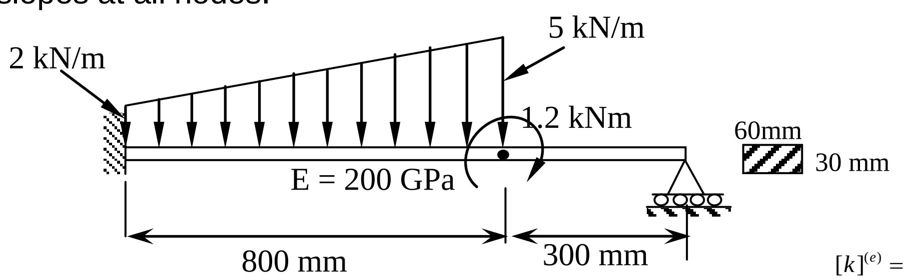



**Fig 2c**

**3A.** The beam structure shown in Fig. 3a is subjected to uniformly varying load and a point moment. Using FEM, obtain the global stiffness relation with minimum number of elements. Apply the boundary conditions as shown and determine the deflections and slopes at all nodes. **05**




$$[k]^{(e)} = \frac{EI}{l^3} \begin{array}{c} \begin{array}{cccc} iy & i\theta & jy & j\theta \end{array} \\ \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \end{array} \begin{array}{c} iy \\ i\theta \\ jy \\ j\theta \end{array}$$

**Fig 3a**

**3B.** Fig 3b shows a stepped bar subjected to body forces and traction forces. Using Rayleigh-Ritz method, find an approximate solution for the axial displacement of the bar. Assume a quadratic polynomial displacement function. Use the details given in table 2. **05**

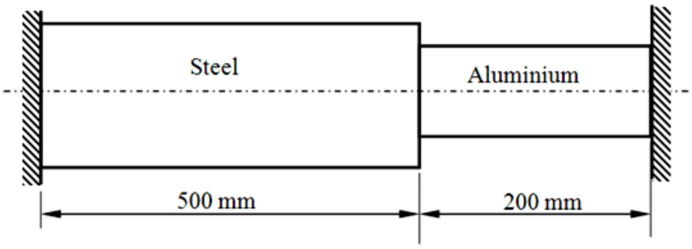



**Fig 3b**

| Table 2 | | |
|---|---|---|
| | Steel | Aluminium |
| Area of cross section | 300 $mm^2$ | 100 $mm^2$ |
| Young's Modulus | 200 GPa | 70 GPa |
| Body force | $10x$ $N/mm^3$ | $5x$ $N/mm^3$ |
| Traction force | $4x$ N/mm | $2x$ N/mm |

**4A.** Show that the Thermal Force vector in an uniaxial inclined bar element due to temperature change ΔT and whose linear thermal coefficient of expansion is α, is given by (with usual notations) **03**

$$\begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \end{Bmatrix}^T = E\,A\,\alpha\,\Delta T \begin{Bmatrix} -C \\ -S \\ C \\ S \end{Bmatrix}$$

**4B** A composite wall consists of three materials, as shown in Fig 4b.The outer temperature is $T_0$= 20°C. Convection heat transfer takes place on the inner surface of the wall with $T_\infty$=800°C and h = 25 W/ $m^2$°C. Determine the temperature distribution in the wall. **04**

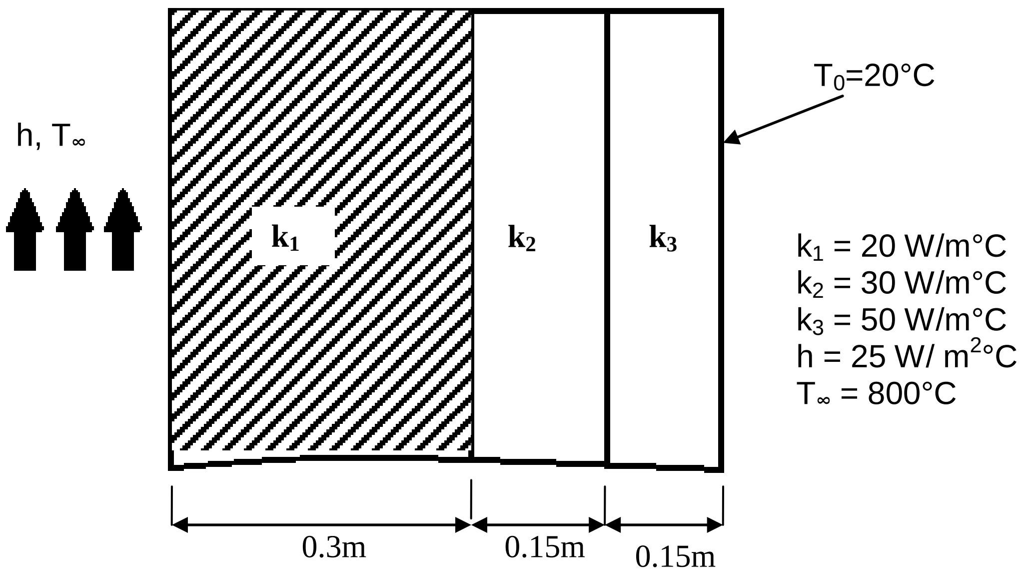



**Fig 4b**

**4C.** For the spring system shown in Fig 4c use minimum potential energy principle to obtain the {F} = [K]{Q}, where, {F} is the force vector, [K] is the stiffness matrix and {Q} is the displacement vector. Also solve for {Q}. **03**

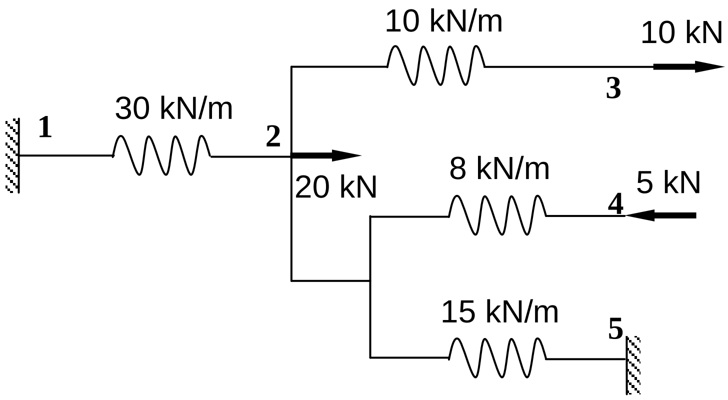



**Fig 4c**

**5A.** The (x, y) coordinates of the corners (nodes) of a triangular element in mm are A(10, 20), B(70,40) & C(30,60). Assuming it to be a CST element, evaluate the global force vector for the element using the following details: Plane Strain Condition, Plate thickness = 5 mm, coefficient of thermal expansion = $12\times10^{-6}/^{o}C$, E= 200 GPa, Poisson's ratio = 0.3, change in temperature = $20^{o}C$. **04**

| Corner | Point Load (N) | |
|---|---|---|
| | X direction | Y direction |
| A | 5000 (rightwards) | 4000 (downwards) |
| B | 10000 (leftward) | 6000 (upward) |
| C | 10000 (rightward) | 13000 (downwards) |

**5B.** Derive the shape functions for 3 noded CST element **03**

**5C.** A cylinder of uniform cross section carrying compressed fluid is subjected to internal pressure on its wall. The pressure inside the cylinder is increased when it is subjected to external heating. The stress on cylinder's wall is analyzed using FEM. **03**

i) Which element type can be used for this analysis? Justify your answer.
ii) Whether element type selected can address temp effects in problem.
iii) If the cross section is non-circular, can the same element type be used?

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