Question Paper

Exam Date & Time: 11-May-2019 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
I SEMESTER B.Sc. (Applied Sciences) - in Engg.
END-SEMESTER THEORY EXAMINATION- APRIL / MAY 2019
MATHEMATICS - 1 [IMA 111]

Marks: 100 **Duration: 180 mins.**

Answer ANY FIVE full Questions.
Missing data, if any, may be suitably assumed

1)

A) If $y = (sin^{-1}x)^2$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + n^2y_n = 0$. Hence find $y(0)$. (7)

B) Find the length of one arc of the cycloid $x = a(\theta - sin\theta), y = a(1 - cos\theta)$ (7)

C) Obtain reduction formula for $\int sin^m x cos^n x\, dx$ where $m, n > 0$ (6)

2)

A) Find the evolute of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ (7)

B) Find the area of a loop of the curve $x^3 + y^3 = 3axy$ (7)

C) Evaluate: (6)

i. $\int_0^{2a} x^3(2ax - x^2)^{\frac{3}{2}}\, dx$

ii. $\int_0^{\infty} \frac{x^2}{(1+x^2)^{\frac{7}{2}}} dx$

3)

A) Show that $\frac{v-u}{1+v^2} < tan^{-1}v - tan^{-1}u < \frac{v-u}{1+v^2}$ whenever $0 < u < v$. Hence deduce $\frac{\pi}{4} + \frac{3}{25} < tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}$ (7)

B) Find the area common to the cardioids $r = a(1 + cos\theta)$ and $r = a(1 - cos\theta)$ (7)

C) Trace the curve $y^2(a - x) = x^2(a + x)$ with explanation. (6)

4)

A) Expand $\log(1 + sinx)$ in powers of x up to term containing x^4 (7)

A)
B) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^2 x^n}{(2n)!}$ (7)

C) Trace the curve whose parametric equation is $x = a\cos^3 t,\ y = b\,sin^3 t$ (6)

5) Find the angle of intersection of the curves $r = 3cos\theta$ and $r = 1 + cos\theta$ (7)
A)
B) Find the equation of the plane that passing through the line of intersection of $2x + y - z = 3$ and $5x - 3y + 4z + 9 = 0$ and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$ (7)

C) Trace the curve $r = a\ sin3\theta$ with explanation. (6)

6) Find the radius of curvature for $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ (7)
A)
B) Find the image of the point $(2, -3, 4)$ with respect to the plane $4x + 2y - 4z + 3 = 0$ (7)

C) Test the convergence of the series (6)

$$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \cdots$$

7) Find the n[th] derivative of the following functions: (7)
A)
i. $x^{n-1} log x$

ii. $\frac{3}{(x+1)(2x-1)}$

B) Find the magnitude and the equation shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ (7)

C) (6)

Evaluate the following:

i. $\lim\limits_{x \to o} \left[\frac{1}{x^2} - \frac{1}{xtanx}\right]$

ii. $\lim\limits_{x \to o} (1 + sinx)^{\cot x}$

8) State Cauchy's mean value theorem and verify the same for the functions (7)
A) $f(x) = x^3 - 3x^2 + 2x$ and $g(x) = x^3 - 5x^2 + 6x$ in $\left(0, \frac{1}{2}\right)$

B) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0,\ x + y + z = 3$ as great circle. (7)

C) Test the convergence of the series $\frac{1!2}{1} + \frac{2!4}{4} + \frac{3!8}{27} + \frac{4!16}{256} + \cdots \infty$. (6)

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