Question Paper

Exam Date & Time: 23-Apr-2019 (02:00 PM - 05:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
II SEMESTER B.Sc. (APPLIED SCIENCES) IN ENGINEERING END SEMESTER EXAMINATION-APRIL/MAY 2019
MATHEMATICS - II [IMA 121 - S2]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.

Missing data if any, Suitably Assumed.

1)

A) Find the extreme values of the function $f(x,y) = xy(a-x-y)$. (7)

B) Change the order of integration and hence evaluate, $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$. (7)

C) Define basis for R^n. Test whether the set $B = \{(1,1,0),(1,0,-2),(1,1,1)\}$ forms a basis for R^3. If so, represent $(1,2,3)$ in terms of basis vectors. (6)

2)

A) Define orthogonal basis of vectors. Using Gram-Schmidt process construct an orthonormal basis from the set of vectors $\{(1,1,1),(2,-1,2),(1,2,3)\}$ $in\ R^n$. (7)

B) Evaluate $\int\int_S \nabla\times\vec{A}.\hat{n}\, ds$, where $\overrightarrow{A} = (y-z+2)\hat{\imath} + (yz+4)\hat{\jmath} - xz\,\hat{k}$ and S is the surface of the cube $x=0, y=0, z=0, x=2, y=2, z=2$ above xy plane. (7)

C) Using Double integral, find the area lying between the parabola $y = x^2$ and the line $y = x$. (6)

3)

A) Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1,-1,2)$. (7)

B) Find the minimum value of $x^2+y^2+z^2$ under the condition $ax+by+cz=p$. (7)

C) Test for consistency and solve (6)

$$5x+3y+7z=4,\ 3x+26y+2z=9,\ 7x+2y+10z=5.$$

4) (7)

A)

Verify stokes theorem for $\vec{A} = (2x-y)\hat{\imath} - yz^2\,\hat{\jmath} - y^2z\,\hat{k}$ where S is the rectangle bounded by the lines x = -a, x = a ,y = 0 and y = b

B) (7)

Define Free and Basic variables. Test for consistency and solve:

$$x + y + z = 4,\ \ 3x + y + z = 2,$$

C) (6)

If $H = f(y-z, z-x, x-y)$, prove that $H_x + H_y + H_z = 0$.

5) (7)

A) Prove $\nabla^2 r^n = n(n+1)r^{n-2}$, where n is a constant.

B) (7)

Verify Green's theorem in the plane for $\oint (xy + y^2)dx + x^2dy$ over the region bounded by $y = x$ and $y = x^2$.

C) (6)

Evaluate $\int_0^\infty \int_0^\infty e^{-x^2}\ e^{-y^2}\,dy\,dx$.

6) (7)

A) Find the Jacobian of Spherical coordinate systems

B) (7)

Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 1 & 1 \\ -1 & 1 & 2 & 2 \\ 2 & 1 & -2 & 1 \\ 1 & 2 & -2 & 2 \end{bmatrix}$ by reducing into Echelon form.

C) (6)

Find the value of the constants a, b and c so that directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(1,2,-1)$ has maximum of magnitude 64 in a direction parallel to z- axis.

7) (7)

A) If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$ show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r).$$

B) (7)

Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integral

C) (6)

Prove that $\sqrt{\pi}\,\gamma(2m) = 2^{2m-1}\,\gamma(m)\gamma\left(m + \frac{1}{2}\right)$.

8) (7)

A) Find inverse of the matrix $\begin{bmatrix} 1 & 1 & 0 \\ 2 & -2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ by using elementary row transformations.

B) (7)

Is $F = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2 k$ is conservative? If so find scalar potential.

C) Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}\, dx \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$. (6)

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