Question Paper

Exam Date & Time: 06-Jun-2019 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTER FOR APPLIED SCIENES
II SEMESTER B.Sc. (APPLIED SCIENCES) IN ENGINEERING END SEMESTER THEORY EXAMINATION - APRIL/MAY 2019

Mathematics - II [IMA 121]

Marks: 100 **Duration: 180 mins.**

Answer 5 out of 8 questions.
Missing data, if any, may be suitably assumed

1)

A) If xyz = 8, find the maximum value of $u = \frac{5xyz}{x+2y+4z}$ (7)

B) Using Gram Schmidt orthogonalization process, construct an orthonormal set of basis vectors, from the given set of vectors (1, 1, 1) (0, 1, 1) (1, 0, 1) (7)

C) Find the inverse of the matrix A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, using elementary row transformations, if it exists. (6)

2)

A) Test whether the following set of vectors form basis of R^3, if so express (1, 2, 3) in terms of basis vectors (1, 1, 0), (1, 0, −2), (1, 1, 1). (7)

B) Solve the system of equations using Gauss elimination method: (7)
x + y + z = 8, 2x + 3y + 2z =19, 4x + 2y + 3z =13

C) Verify Green's theorem in the plane for $\int_C (xy + y^2)\,dx + x^2dy$ where C is the closed curve of the region bounded by y = x and y = x^2. (6)

3)

A) Using Gauss divergence theorem evaluate $\iint_s F \cdot nds$ where F= 4xzi − y^2j + yzk and S is the surface of the cube bounded by x = 0, x = 2, y = 0, y =2, z = 0, z = 2 (7)

B) Evaluate $\int_0^\infty \int_0^\infty e^{-x^2} e^{-y^2} dy\, dx$. (7)

C) Find the area enclosed between the curves $y^2 = 4ax$ and $x^2 = 4ay$. (6)

4)

A) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \sqrt{x^2+y^2}$dydx by transforming to polar coordinates. (7)

B) If $u = f(r)$ where $r = \sqrt{x^2+y^2}$ show that (7)
$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

C) Show that $A = (6xy + z^3)\,i + (3x^2 - z)\,j + (3xz^2 - y)\,k$ is irrotational. Find ϕ such that $A = \nabla\phi$. (6)

5) A) If $H = f(y - z, z - x, x - y)$ prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$ (7)

B) Evaluate $\iint A \cdot n\,ds$ where $A = 18zi - 12j + 3yk$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant. (7)

C) Prove $\nabla^2 r^n = n(n+1)r^{n-2}$, where n is a constant. (6)

6) A) If R is the region bounded by the circle $x^2 + y^2 = 1$ in the first quadrant, evaluate $\iint_R \frac{xy}{\sqrt{1-y^2}}\,dy\,dx$ (7)

B) Change the order of integration and evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$ (7)

C) Prove the relation between Beta and Gamma function $\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$. (6)

7) A) . State and prove Green's theorem in xy- plane. (7)

B) If the kinetic energy T is given by $T = \frac{1}{2} m v^2$ find approximately the change in T as the mas changes from 49 to 49.50 and the velocity v changes from 1600 to 1590. (7)

C) If $z = \log(x^2 + xy + y^2)$ show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2$ (6)

8) A) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using double integration (7)

B) Test for consistency and solve: (7)

$$x_1 - 2x_2 + 4x_3 = 9$$

$$x_1 - x_2 + 2x_3 = 6$$

C) Find $\frac{dy}{dx}$ if $x^y - y^x = c$ using partial derivatives (6)

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