Question Paper

Exam Date & Time: 06-May-2019 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENES
III SEMESTER B.Sc.(Applied Sciences) IN ENGINEERING END SEMESTER THEORY EXAMINATION-APRIL/MAY 2019
MATHEMATICS - III [MA 231]

Marks: 100 **Duration: 180 mins.**

Answer ANY FIVE full Questions.
Missing data, if any, may be suitably assumed

1)

A) Solve: $\frac{dy}{dx} = \cos(x+y+1)$. (7)

B) Solve by the method of separation of variables $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} = 0$. (7)

C) Find the inverse Laplace transform of $\frac{4s+5}{(s-1)^2(s+2)}$. (6)

2)

A) Solve: $(3x-y+4)dy = (6x-2y-7)dx$. (7)

B) Solve $\frac{dy}{dx} = x^2y - 1$ given $y(0) = 1$ and find y at x = 0.1 and x = 0.2 to five decimal places taking h = 0.1 by Taylor's series method (carry up to fourth order derivatives). (7)

C) Apply Convolution theorem to evaluate $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+9)}\right]$. (6)

3)

A) Solve: $(x^3+y^3+6x)dx + y^2x\,dy = 0$. (7)

B) Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0)=1$ by Euler's method, for x = 0.1 taking h = 0.02. (7)

C) Solve $y'' + 6y' + 9y = 12t^2e^{-3t}$, $y(0) = y'(0) = 0$ using Laplace transform method. (6)

4)

A) Solve: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-x}\sin x$. (7)

A) $\quad d x^2 \quad d x$

B) Given $\frac{dy}{dx} = x + y^2$, $y(0) = 1$. Compute y at x = 0.1 and x = 0.2 by taking h = 0.1 using Runge - Kutta method of order four. (7)

C) Given that $f(z) = u + i\,v$ is analytic and $v(x, y) = -\sin x \sin hy$. Show that $v(x, y)$ is harmonic. Find the conjugate harmonic of $v(x, y)$. (6)

5)

A) Find the Laplace transform of $f(t) = e^{3t+7} + 4\sin^2 3t + 5\cos 6t \cos 4t$. (7)

B) Given that $f(z) = u + iv$ is analytic and $u(r, \theta) = \left(r - \frac{1}{r}\right)\sin\theta,\ r \neq 0$. (7)

Find the orthogonal trajectories of the family of curves, u = c, a constant.

C) Solve $y'' - 2y' + y = \frac{e^x}{x^5}$ using method of variation of parameters. (6)

6)

A) Evaluate the integral $\int_0^\infty te^{-3t}\cos t\,dt$ using Laplace transform. (7)

B) Solve: $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin(\log(1+x))$. (7)

C) Evaluate $\oint_C \frac{dz}{z^2+9}$, where C is $|z + 3i| = 2$, using Cauchy's integral formula. (6)

7)

A) Solve: $\frac{dx}{dt} + 2y = -\sin t$; $\frac{dy}{dt} - 2x = \cos t$. (7)

B) Find the Laplace transform of function $f(t) = \begin{cases} t, & 0 < t \leq \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$ (7)

and $f(t + 2\pi) = f(t)$.

C) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region 1 < |z| < 2. (6)

8)

A) Solve $u_{xx} + 2u_{xy} + u_{yy} = 0$ using the transformation $v = x,\ z = x - y$. (7)

B) Rewrite $f(t) = \begin{cases} 1, & 0 \leq t < 3 \\ t, & 3 \leq t < 6 \\ t^2, & t \geq 6. \end{cases}$ in terms of unit step function and hence find its Laplace transform. (7)

C) Using Cauchy's Residue theorem, evaluate $\oint_C \frac{z^2}{(z-1)^2(z+2)} dz$, where C is the circle $|z| = 2.5$. (6)

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