MANIPAL ACADEMY of HIGHER EDUCATION
(Deemed to be University under Section 3 of the UGC Act, 1956)

Reg. No.									

DEPARTMENT OF SCIENCES, II SEMESTER M.Sc
(Applied Mathematics and Computing)
END SEMESTER EXAMINATIONS, APRIL/MAY 2019

Complex Analysis [MAT 4204]

(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours | Date: 25 – 04 - 2019 | MAX. MARKS: 50

Note: (i) Answer **ALL** questions . (II)All questions carry equal Mark **(3 + 3 + 4)**

1A. If f(z) = u + iv is analytic then show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f\,'(z)|^2$

1B. Find the analytic function f(z) = u + iv if u = e^x(x cosy – y siny)

1C. Find the bilinear transformation which maps the points 1, i , -1 onto the points i, 0, -i. Also find (i) image of |z| < 1 (ii) fixed points of the transformation.

2A. Show that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

2B. Show that the line integral $\int_\gamma \; pdx + qdy$, defined in a region Ω, depends only on the end points of γ if and only if there exists a function U(x, y) in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p, \frac{\partial U}{\partial y} = q.$

2C. If a function f(z) is analytic on a rectangle R, then show that $\int_{\partial R} f(z)dz = 0$, where ∂R is the boundary of R.

3A. If a piecewise differentiable closed curve γ does not pass through the point a, then the show that the value of the integral $\int_\gamma \frac{dz}{z-a}$ is a multiple of 2πi.

3B. Using residues evaluate $\int_{|z|=3} \frac{z^2}{(z-1)^2(z+2)}dz$

3C. Obtain all possible expansions of $f(z) = \frac{1}{(z-1)(z-2)}$, about the origin.

4A. State and prove Argument principle

4B. State and prove Lucas's theorem

4C. Evaluate $\int_0^{2\pi} \frac{d\theta}{37-12cos\theta}$.

5A. Obtain Cauchy's estimate for derivatives of a function f(z) which is analytic and bounded on C: $|z - a| = R$ and hence prove Liouville's theorem.

5B. Evaluate $\int_0^{\infty} \frac{cosmx}{a^2+x^2} dx$ using contour integration.

5C. Suppose that $|f(z)| \leq |f(z_0)|$ at each point z in some neighbourhood $|z - z_0| < \varepsilon$, in which f(z) is analytic. Then show that f(z) is constant in the neighbourhood. Hence establish the maximum modulus principle.

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