MANIPAL ACADEMY of HIGHER EDUCATION
(Deemed to be University under Section 3 of the UGC Act, 1956)

Reg. No.									

DEPARTMENT OF SCIENCES, II SEMESTER M.Sc
(Applied Mathematics and Computing))
END SEMESTER EXAMINATIONS, April/May 2019

Subject [Linear Algebra-MAT 4206]

(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date: 27.04.2019 MAX. MARKS: 50

Note: (i) Answer all **FIVE FULL** questions
(ii) All questions carry equal marks (4+3+3)

1. (a) If A is a $m \times n$ matrix with entries in a field F, then show that $rowrank(A) = column\ rank(A)$.
   (b) Check whether the following matrix is diagonalizable; $\begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$.
   (c) Let e be an elementary row operation and let E be the $m \times m$ elementary matrix $E = e(I).where$ I is the identity matrix of order m. Then, show that for every $m \times n$ matrix A, $e(A) = EA$.

2. (a) Let $W_1$and W_2be subspaces of a finite-dimensional vector space V. Then with usual notation, show that
   (i) $(W_1 + W_2)^0 = W_1^0 \cap W_2^0$.
   (ii) $(W_1 \cap W_2)^0 = W_1^0 + W_2^0$.
   (b) If A is a $n \times n$ triangular matrix, then show that $\det(A)$ is the product of diagonal entries of A.
   (c) Let V be a real vector space and E be an idempotent linear operator on V, i.e., a projection. Prove that $(I + E)$ is invertible and find $(I + E)^{-1}$.
3. (a) If A is an $n \times n$ matrix, the prove that, the following are equivalent.
   (i) A is invertible
   (ii) The homogeneous system $AX = 0$ has only the trivial solution $X = 0$.
   (iii) The system of equations $AX = Y$ has a solution $X$for each $n \times 1$ matrix Y.
   (b) If V is an inner product space, then for any vectors α, β in V, show that
   (i) $|< \alpha, \beta >| \leq \|\alpha\| \, \|\beta\|$.
   (ii) $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$.

(P.T.O)

(c) Let T be a linear transformation from vector space V into W. Define T^t. Show that the null space of T^t is the annihilator of T. Also, show that, if V and W are finite dimensional, then $rank(T) = rank(T^t)$.

4. (a) Let $n > 1$ and let D be an alternating $(n-1)$-linear function on $(n-1) \times (n-1)$ matrices over K. Prove that for each $j, 1 \le j \le n$, the function $E_j$defined by $E_j(A) = \sum_{i=1}^{n} (-1)^{i+j} A_{ij} D_{ij}(A)$ is an alternating n- linear function on $n \times n$ matrices. Also prove that if D is a determinant function, so is each E_j.

(b) If f is a non-zero linear functional on the vector space V, then show that the null space of f is a hyperspace in V. Conversely, show that every hyperspace in V is the null space of a (not unique) non-zero linear functional on V.

(c) For a square matrix A, show that sum of its eigenvalues is trace of A and product of the eigenvalues is determinnat of A.

5. (a) Let R be the field of real numbers, and let D be a function on 2×2 matrices over R, with values in R, such that $D(AB) = D(A)D(B)$ for all A, B. Suppose also that $D\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) \neq D\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$. Prove the following.

(i) $D(0) = 0$;
(ii) $D(A) = 0$ if $A^2 = 0$;
(iii) $D(B) = -D(A)$ if B is obtained by interchanging the rows (or columns) of A.
(iv) $D(A) = 0$ if one row of A is zero.

(b) Let V be a finite-dimensional vector space over the field F and let Tbe a linear operator on V. Then prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1) \dots (x - c_k)$ where $c_1, \dots, c_k$ are distinct elements of F.

(c) Is there a linear transformation T from R^3 into R^2such that $T(1, -1, 1) = (1, 0)$ and $T(1, 1, 1) = (0, 1)$? If exists, find one of them. How many such transformations exist?

****