MANIPAL ACADEMY of HIGHER EDUCATION
(Deemed to be University under Section 3 of the UGC Act, 1956)

Reg. No.										

DEPARTMENT OF SCIENCES, IV SEMESTER M.Sc (Applied Mathematics and Computing)
END SEMESTER EXAMINATIONS, APRIL 2019

SUBJECT: Stochastic Processes and Reliability [MAT 5202]

(REVISED CREDIT SYSTEM-2017)

Time: 3 Hours Date: 24.04.2019 MAX. MARKS: 50

Note: (i) Answer **ALL** questions. (ii) All questions carry equal Marks **(3 + 3+ 4)**
(iii) Draw diagrams and write equations wherever necessary

1A. A process moves on the integers 1, 2, 3, 4 and 5. It starts at 1 and on each successive step, moves to an integer greater than its present position, moving with equal probability to each of the remaining larger integers. State five is an absorbing state. Find (i)t.p.m (ii) expected number of steps to reach state five.

1B. Define Generating function. Let X be a random variable denoting the number of tosses required to get two consecutive heads when a fair coin is tossed. Show that p.g.f of X is $[(s^2/4)\,\{1 - s/2 - (s/2)^2\}^{-1}]$.

1C. Let Yn= $a_0X_n + a_1X_{n-1}$ (n = 1, 2,….), where a_0, a_1 are constants and X_n(n=0,1, 2.......) are i.i.d.random variables with mean 0 and variance σ^2. Is $\{Y_n, n \geq 1\}$ covariance stationary? Is it a Markov process?

2A. Divide the interval [0, t] into a large number n of small intervals of length h and suppose that in each small interval, Bernoulli trials with probability of successes λh are held. Show that the number of successes in an interval of length t is a Poisson process with mean λt. State the assumptions you make.

2B. Consider a Markov chain with three states, S= {1, 2, 3} that has the following

transition matrix $\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$

i) Draw the state transition diagram for this chain and possible in trees.

ii) If we know P(X1=1) =P(X1=2) =1/4 find P(X1=3, X2=2, X3=1).

2C. Consider two independent series of events E and F occurring in accordance with Poisson processes with mean 3t and 5t respectively. Prove that the number N of occurrences of E between two successive occurrences of F has geometric distribution.

3A. State and derive Yule – Fury birth process.

3B. Suppose that a ball is thrown at random to one of the r-cells. Let $X_n(n\geq 1)$ be said to be in state k (where k=1,2,…..,r) if after n throws k cells are occupied. Find P and P^2. Is P irreducible when r=3?

3C Let $\{X_n, n\geq 0\}$ be a Markov chain with four states 1, 2, 3 and 4 with transition matrix. $P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ Evaluate F_{22} and μ_{22}.

4A. Consider a sequence $\{X_n\}$ of independent coin-tossing trials with probability p for head H in a trial. Denote the states of X_n by states 1, 2, 3, 4 according as the trial numbers (n-1) and n result in HH, HT, TH and TT respectively. Show that $\{X_n\}$ is a Markov chain. Find (i) P^{901} (ii) P ($X_3 = 3 /\ X_1 = 4$).

4B. Define the followings:
(i) Immigration – Emigration Process (ii) Renewal process
(iii) Time dependent Poisson Process

4C. Prove that in an irreducible chain, all states are of same type. They are either all transient, all persistent null, or all persistent non null, all states are aperiodic or periodic and in the latter case they all have the same period.

5A. If $N_1(t),N_2(t)$ are two independent Poisson processes with parameters λ_1 and λ_2 respectively, then find the distribution of $P\{ N_1(t) =k \mid N_1(t) + N_2(t)=n.\}$. Hence write mean and variance.

5B **i)** Solve: $P_{n+2} + 2P_{n+1} + P_n = 0$, using generating function.

ii)If passengers arrive at a taxi stand in accordance with a Poisson process with parameter 3 and taxis arrive to the stand in accordance with a Poisson process with parameter 5 then find average number of excess of passengers over taxis in 3 hours and what is its distribution? Compute

5C. Find the probability of ultimate extinction in case of linear growth process starting with i individuals initially.