MANIPAL
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Reg. No. | | | | | | | | | | |

DEPARTMENT OF SCIENCES, M. Sc. (Physics)
II SEMESTER, END SEMESTER EXAMINATIONS
APRIL 2019
Subject: Quantum Mechanics II (PHY-4206)
(REVISED CREDIT SYSTEM - 2017)

Time: 3 Hours   Date: April 2019   MAX. MARKS: 50

Note: (i) Answer all the questions.
(ii) Answer the questions to the point.

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1. (i) The vector $\vec{J}$ gives the sum of angular momentum $\vec{J_1}$ and $\vec{J_2}$. Prove that

$$[J_x, J_y] = i\hbar J_z, \quad [J_y, J_z] = i\hbar J_x, \quad [J_z, J_x] = i\hbar J_y$$

Is $\vec{J_1} - \vec{J_2}$ an angular momentum? [5]
(ii) Optimize the trial function $exp(-\alpha r)$ and evaluate the ground state energy of the hydrogen atom. [5]
2. (i) Solve the following one dimensional infinite potential well:

$$V(x) = 0 \quad \textbf{for} - a < x < a; \quad V(x) = \infty \quad \textbf{for} |x| > a$$

using the WKB method and compare it with the exact solution. [3]
(ii) Mention the validity condition of the WKB method. [2]
(iii) By applying the WKB method connection formulae obtain expression for energy levels of a particle in a potential well. [5]
3. (i) Briefly discuss the time dependent perturbation theory for a two level system. [5]
(ii) A system is in an unperturbed state n is suddenly subjected to a constant perturbation $H'(r)$ which exists during time $0 \to t$. Find the probability for transition from state n to state k and show that it varies simple harmonically with
angular frequency= $\frac{E_k - E_n}{2\hbar}$ **and amplitude=** $4\frac{|H'_{kn}|^2}{(E_k-E_n)^2}$**.** [5]
4. (i) Using first order Born approximation for weak and spherically symmetric potential obtain expression of scattering amplitude. [5]

(ii) In a scattering experiment, the potential is spherically symmetric and the particles are scattered at such energy that only s and p waves need to be considered.
(a) Show that the differential cross-section $\sigma(\theta)$ can be written in the form $\sigma(\theta) = a + bcos\theta + ccos^2\theta$.
(b) What are the values of a, b, and c in terms of phase shifts?
(c) What is the value of total cross-section in terms of a, b, c? [5]
5. (i) What are the different approaches of relativistic quantum mechanics? What are the limitations of the Klein Gordon equation? [4]
(ii) Show that the following matrices form a representation of Dirac's matrices:
$\alpha_x = \begin{pmatrix} \sigma_x & 0 \\ 0 & -\sigma_x \end{pmatrix}$, $\alpha_y = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$, $\alpha_z = \begin{pmatrix} \sigma_z & 0 \\ 0 & -\sigma_z \end{pmatrix}$, $\beta = \begin{pmatrix} 0 & iI \\ -iI & 0 \end{pmatrix}$. [6]

Useful formulae:

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

$$\int_0^\infty x^n exp(-ax)\,dx = \frac{n!}{a^{n+1}}, \quad \textbf{where} \quad n \geq 0, \quad a > 0$$

Barrier to the right of the turning point

$$\frac{2}{\sqrt{k}}cos\left(\int_x^{x_1} k\,dx - \frac{\pi}{4}\right) \longleftarrow \frac{1}{\sqrt{\gamma}}exp\left(-\int_{x_1}^{x} \gamma\,dx\right)$$

$$\frac{1}{\sqrt{k}}sin\left(\int_x^{x_1} k\,dx - \frac{\pi}{4}\right) \longrightarrow -\frac{1}{\sqrt{\gamma}}exp\left(\int_{x_1}^{x} \gamma\,dx\right)$$

Barrier to the left of the turning point

$$\frac{1}{\sqrt{\gamma}}exp\left(-\int_x^{x_2} \gamma\,dx\right) \longrightarrow \frac{2}{\sqrt{k}}cos\left(\int_{x_2}^{x} k\,dx - \frac{\pi}{4}\right)$$

$$-\frac{1}{\sqrt{\gamma}}exp\left(\int_x^{x_2} \gamma\,dx\right) \longleftarrow \frac{1}{\sqrt{k}}sin\left(\int_{x_2}^{x} k\,dx - \frac{\pi}{4}\right)$$