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MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

SECOND SEMESTER M.TECH. (CONTROL SYSTEMS)

END SEMESTER DEGREE EXAMINATION, APRIL/MAY - 2019

SUBJECT: ROBUST & H∞ CONTROL [ICE 5249]

TIME: 3 HOURS MAX. MARKS: 50

Instructions to candidates :*Answer ALL questions and missing data may be suitably assumed.*

1A Is it possible to find the norm of the system $\boldsymbol{G(S)} = \begin{bmatrix} \frac{\mathbf{1}}{\boldsymbol{s-1}} & \mathbf{0} \\ \mathbf{0} & \frac{\mathbf{10}}{\boldsymbol{s+1}} \end{bmatrix}$? Justify your answer.

1B Find $\|\ \|_\infty$ and $\|\ \|_2$ norms of $\mathbf{G(s)} = \frac{\mathbf{1}}{\mathbf{s+\alpha}} - \frac{\mathbf{2}}{\mathbf{s+2\alpha}}$ as a function of real parameter **'α'**.

1C What do you mean by **2**-norm and ∞-norm of the system? Find the 2-norm of the system $\mathbf{G(s)} = \begin{bmatrix} \frac{2}{s+10} \\ \frac{20}{s+1} \end{bmatrix}$.

(2+3+5)

2A Show that set of all controllers of plant $\boldsymbol{P} \in \boldsymbol{R\mathcal{H}_\infty}$ for which the feedback system shown in Fig. **Q2A** is internally stable is given by $\boldsymbol{C} = \left\{\frac{\boldsymbol{Q}}{\boldsymbol{1-PQ}} : \boldsymbol{Q} \in \boldsymbol{R\mathcal{H}_\infty}\right\}$.

2B Find $\frac{e}{r}$, $\frac{y}{d}$ and $\frac{y}{r}$ ratios and illustrate the sensitivity transfer function $S(s)$ and complementary sensitivity transfer function $T(s)$ for the unity feedback system as shown in Fig. **Q2A**. Also, show that $\bar{\boldsymbol{\sigma}}(\boldsymbol{S(j\omega)}) + \bar{\boldsymbol{\sigma}}(\boldsymbol{T(j\omega)}) \geq \mathbf{1}, \forall \boldsymbol{\omega}$.

2C Prove that feedback system shown in Fig. **Q2A** is internally stable if and only if the two conditions: a) the transfer function $\mathbf{1} + \boldsymbol{PCF}$ has no zeros in $\boldsymbol{Res} \geq \mathbf{0}$ and, b) there is no pole-zero cancellation in $\boldsymbol{Res} \geq \mathbf{0}$ when product $\boldsymbol{PCF}$ is formed.

(2+3+5)

3A Consider the standard feedback loop shown in Fig. **Q2A,** where $P(s) = \frac{1}{s^2-1}$ $C(s) = \frac{s-1}{s+1}$ and $F(s) = 1,\ n = d = 0$. Is the feedback loop stable? Justify your answer.

3B Assume that unity feedback system as shown in Fig. **Q2A** is internally stable and $n = \boldsymbol{d} = \mathbf{0}$. Show that if input $\boldsymbol{r(t)}$ is the unit step then $\boldsymbol{e(t)} \to \mathbf{0}$ as $\boldsymbol{t} \to \infty$ if and only if sensitivity transfer function $\boldsymbol{S(s)}$ has at least one zero at the origin.

3C Derive robust performance condition for the feedback control system having additive uncertainty plant model.

(2+3+5)

4A Define all-pass and minimum phase transfer function with examples.

4B Derive robust stability condition for the feedback system shown in Fig. **Q4B.**

4C State and prove small gain theorem with neat diagrams.

(2+3+5)

5A Consider nominal plant $\boldsymbol{P(s)} = \frac{\mathbf{1}}{\boldsymbol{s+a_0}}$ and actual plant model $\widetilde{\boldsymbol{P}}(\boldsymbol{s}) = \frac{\mathbf{1}}{\boldsymbol{s+a}}$ **with** $\boldsymbol{a = a_0 + \delta\delta_m}, |\boldsymbol{\delta}| \leq \mathbf{1}$. Assume suitable weighting function $\boldsymbol{W}$ and rewrite uncertain system in the form $\widetilde{\boldsymbol{P}} = \boldsymbol{P}(\mathbf{1} + \Delta\boldsymbol{W})^{-\mathbf{1}}$, where Δ is stable transfer function given by $\Delta = \boldsymbol{\delta}$ with $|\boldsymbol{\delta}| \leq \mathbf{1}$.

5B Compute an internally stabilizing controller for the given plant model $\boldsymbol{G(s)} = \frac{\mathbf{1}}{(\boldsymbol{s}+\mathbf{1})(\boldsymbol{s}+\mathbf{2})}$ in the unity feedback system so that output asymptotically tracks a ramp input.

5C For the given weighting functions $\boldsymbol{W_\Delta}$ and $\boldsymbol{W_P}$, derive the conditions on loop function $\boldsymbol{L}$ for the regions $\boldsymbol{A}, \boldsymbol{B}$ and $\boldsymbol{C}$ as shown in the **Fig. Q5C** to satisfy robust performance inequality $\left\| |\boldsymbol{W_p S}| + |\boldsymbol{W_\Delta T}| \right\|_\infty < \mathbf{1}.$

(2+3+5)

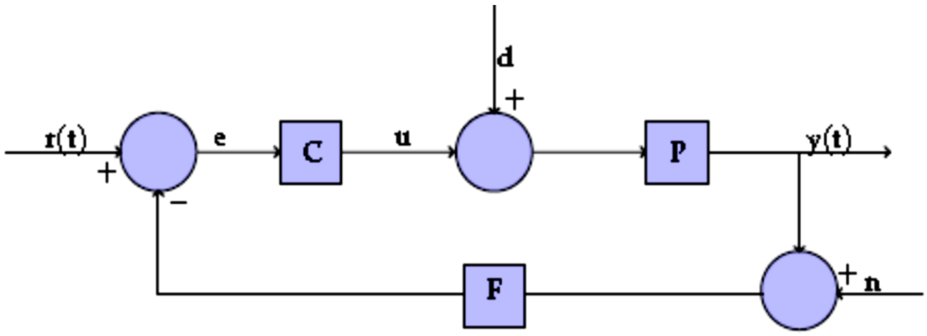

Fig. Q2A

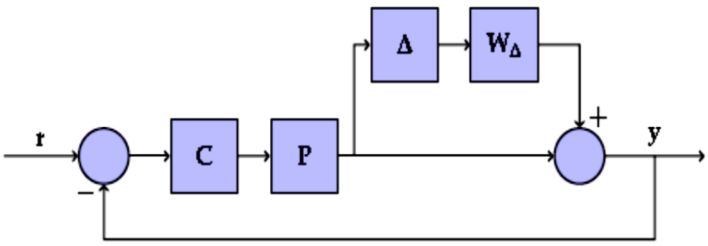

Fig. Q4B

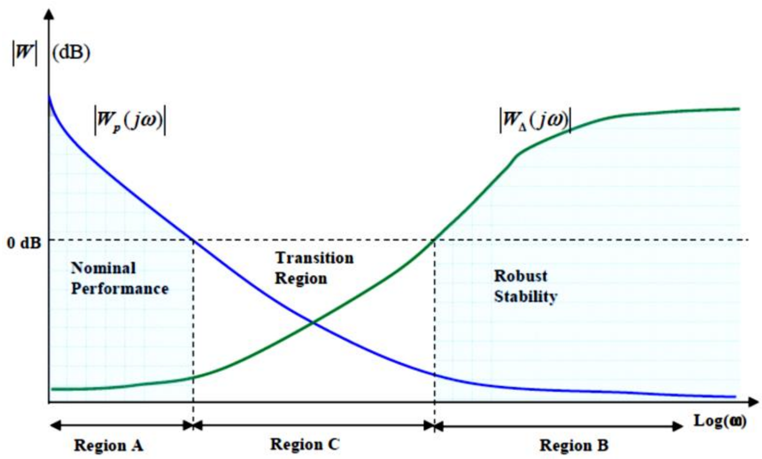

Fig. Q5C

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